Article

Local Pressure of the Gravity Field & Vacuum

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Abstract

The existence of negative pressure of vacuum follows from the cosmological models, based on the results of observations. But, is it possible to detect the pressure of the vacuum as per the geometry of the space around the local gravity system? The gravitational defect of mass is interpreted as the transfer of energy to the vacuum, which becomes apparent from its deformation. We determine the gravitational impact of matter on the vacuum and opposite in the sign pressure of it in case of weakly gravitating static centrally symmetric distribution of matter. We have evaluated a possibility to extend the obtained results to arbitrary gravitational systems. The equation of state gives the deceleration parameter of the universe consistent with its accelerating expansion.

Keywords: Symmetric distribution, pressure, gravitational field, vacuum, deceleration parameter.

1. Introduction

The non-zero vacuum pressure is an element of cosmological models [1-3], resulting from the solution of Einstein's equations. He proposed that curvature of space-time is responsible for gravity. The gravitational mass of bodies placed in confined volume, is less than the sum of the gravitational masses of these bodies, dispersed over infinite distance. The matter, located more compactly, distorts the space in the local domain in a greater degree, however, creating smaller gravitational mass in comparison with the same amount of matter, distributed over a greater volume [1, 2]. This phenomenon is explained by transfer of energy into the gravitational field, which results in the deformation of vacuum. Accumulation of energy during deformation demonstrates its elasticity. We will take these properties of gravity into consideration, while determining the vacuum pressure.

2. Spherical Source

2.1. Solution of Einstein equations

We analyze a centrally symmetrical static gravitational field (using units with $c = 8\pi G = 1$). In spherical coordinates $x^i = (t, r, \theta, \varphi)$ it is described by the metric:

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$$ds^2 = e^{\nu(r)}dt^2 - e^{w(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

where - and ν, w - are functions of radial coordinate. The centrally symmetric stress-energy tensor T_i^i corresponds to the source of gravitation, which creates this type of field. Solution of Einstein's equations for a spherical body with a radius of a yields [1] the required functions:

$$\nu = -w = \ln\left(1 - \frac{1}{r}\int_0^a T_1^1 y^2 dy\right), r > a \tag{2}$$

$$w = -\ln\left(1 - \frac{1}{r}\int_0^r T_1^1 y^2 dy\right), r \le a, \tag{3}$$

where T_1^1 is the density of energy and variable y has the dimension of length. For the static source of gravitation it is equivalent to the density of matter: $T_1^1 = \rho$.

In the external area the obtained functions ν, w correspond to the Schwarzschild metric, and therefore the value

$$M = 4\pi\int_0^a \rho r^2 dr \tag{4}$$

is the gravitational mass of a spherical body with of radius a. Integration is performed here in case of the element of volume $dV_c = 4\pi r^2 dr$, which corresponds to the coordinate frame, whereas in its proper frame the given element of space volume will be $dV_p = 4\pi r^2 e^{w/2} dr$. Since w has a positive value, it means that the gravitational mass of bodies located in the confined volume is less than the sum of individual gravitational masses of these bodies. This interprets as the transfer of energy, as a source of gravitational field, to the vacuum.

2.2. Case of weak gravitation

The volume of spherical body in proper frame is obtained by integration of elements dV_p with (2) and amounts to

$$V_{int}^p(a) = \int_0^a 4\pi r^2 e^{w/2} dr = \int_0^a 4\pi r^2 \left(1 - \frac{1}{3}\rho r^2\right)^{-1/2} dr. \tag{5}$$

For small space curvature inside the sphere, i.e. with $\rho a^2 << 1$, representation of the expression under the integral into a formal power series turns out to be

$$V_{int}^p(a) = \frac{4\pi}{3}a^3 + \frac{2\pi}{15}\rho a^5. \tag{6}$$

Since the density of matter is constant, the mass of body in this frame or the proper mass will be $M^p = \rho V_{int}^p(a)$. A proper energy of static source of gravitation is defined as $E^p = M^p$.

The gravitational impact on the vacuum is determined as the relation of difference between proper energies of two spherical bodies with identical gravitational mass to the change of proper volume of space. With constant densities ρ_1, ρ_2 and radii a_1, a_2, ($a_1 < a_2$) this mass is $M = (4/3)\pi\rho_1 a_1^3 = (4/3)\pi\rho_2 a_2^3$. The difference of proper masses of two bodies is written as follows:

$$\Delta M^p = M_1^p - M_2^p = \frac{2\pi}{15} a_1^6 \rho_1^2 \left(\frac{1}{a_1} - \frac{1}{a_2} \right). \quad (7)$$

Due to equality of gravitational masses of both bodies, the space distortion in the area $r > a_2$, created by them, will be identical. Let's find the difference between the volumes in the proper frame, which are set in coordinate frame by the condition $r \le a_2$. This volume for the first body is the sum of this body's own volume and the peripheral area $a_1 < r \le a_2$, namely,

$$V_1^p = V_{int}^p(a_1) + V_{ext}^p(a_1, a_2), \quad (8)$$

where the second term is given by

$$V_{ext}^p(a_1, a_2) = \int_{a_1}^{a_2} 4\pi r^2 e^{w/2} dr. \quad (9)$$

Breaking the expression under integral into the formal power series, in case of $M/r << 1$ we obtain

$$V_{ext}^p(a_1, a_2) = \frac{4}{3}\pi(a_2^3 - a_1^3) + 2\pi M(a_2^2 - a_1^2). \quad (10)$$

As a result, the volume (8) will amount to

$$V_1^p = \frac{4}{3}\pi a_2^3 + \frac{1}{15}\pi\rho_1 a_1^3 \left(5a_2^2 - 3a_1^2\right). \quad (11)$$

The area $r \le a_2$ restricts the second body, whose proper volume for the weak gravitational field according to (6) is

$$V_2^p = V_{int}^p(a_2) = \frac{4}{3}\pi a_2^3 + \frac{2}{15}\pi\rho_2 a_2^5. \quad (12)$$

The difference between the proper volumes, confined within the radius a_2 in coordinate frame, will be

$$\Delta V^p = V_1^p - V_2^p = \frac{1}{5}\pi\rho_1 a_1^3 (a_2^2 - a_1^2). \quad (13)$$

The ratio of change in the energy of the spherical body $\Delta E^p = \Delta M^p$ to the change of its volume for small $\Delta a = a_2 - a_1$ retaining its gravitational mass taking (7) into consideration yields

$$\wp = \frac{\Delta E^p}{\Delta V^p} = \frac{1}{3}\rho. \quad (14)$$

Provided that $\wp$ corresponds to the pressure, this expression coincides with the equation of the state of photon gas [4]. Positive pressure of gravity field characterizes the gravitational impact of matter on the vacuum, which lies in its constraint. Accordingly, field pressure upon vacuum is compensated by pressure of the vacuum:

$$p_v = -\wp \quad (15)$$

and one may be considered as mean vacuum pressure in case of weak gravitation inside the static sphere.

3. Source with Isotropic Pressure

Let us examine an arbitrary space-time, containing a source of gravitation with density ρ, which is described by the metric $ds^2 = g_{ij}dx^i dx^j$. We allocate a small area, in which metrical coefficients and density can be considered as constant in the first approximation, pressure is isotropic, and whose boundary is a sphere in the proper frame. The gravity, created by this ball, is described by metric (1).

The metrical coefficients of the space-time without a source of gravitation in this sphere will be slightly different from g_{ij}. The transition to a locally inertial system [1] with the beginning in the point x_0^k is made for the changed metrics using the transformation

$$x'^k = x^k + \frac{1}{2}\left(\Gamma_{ij}^k\right)_{x^l = x_0^l} x^i x^j \tag{16}$$

with Christoffel's symbols Γ_{ij}^k. In this locally flat space we place the absent source of gravitation in the empty sphere. This one will be comply to conditions under which pressure of gravitational field was obtained (14). In case of stationary space-time, the proper pressure of vacuum is determined according to (15) and will be

$$p_v = -\frac{1}{3}\rho. \tag{17}$$

4. Cosmological Parameters

Standard cosmological models propose existence of some positive energy density or dark energy [5,6], which can be interpreted as energy of vacuum. In ΛCDM cosmological model the source of gravitation is stationary in comoving coordinates, which are locally geodesic system to meet the required condition. Equation of state (17) corresponds to stationary space-time but expansion of the universe is accelerating [7], which means $-p_v > \wp$. However, the relative velocity of the universe expansion is equal to Hubble parameter H, which is small at present period, and this equation of state is suitable. So, the pressure of vacuum universe is negative and proportional to the sum of density of its energy, matter and relativistic particles located within it:

$$p_v = -\frac{1}{3}(\rho_\Lambda + \rho_m + \rho_{rel}). \tag{18}$$

Substituting the resulting pressure of vacuum in the Friedmann equations in case of the flat space

$$\rho_c = \frac{3\dot{R}^2}{R^2}, \tag{19}$$

$$p_v = -\frac{2\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} + \rho_\Lambda \tag{20}$$

with critical density ρ_c and a length scale factor R yields

$$\rho_\Lambda = 2\frac{\ddot{R}}{R}. \qquad (21)$$

Introducing the deceleration parameter q_0 we express it through dark energy density parameter as

$$q_0 = -\frac{3}{2}\Omega_\Lambda. \qquad (22)$$

The Planck Collaboration cosmological parameters [8] implies $\Omega_\Lambda =$ 0.6911 ± 0.0062 that gives $q_0 = -1.03665 \pm 0.093$. Evidence for accelerating expansion of the universe came from he supernova observation and the deceleration parameter is estimated [7] as $q_0 = -1 \pm 0.4$ within some other model under condition $q_0 = \frac{\Omega_m}{2} - \Omega_\Lambda$.

5. Discussion

Transfer of obtained result for the weakly gravitating sphere into the arbitrary point of space-time requires observation of the strong principle of equivalence, according to which laws of gravitation, tested in a freely moving laboratory, are independent of its velocity and location in space-time. Landau and Lifshitz wrote [1] about equation $p = (1/3)\rho$ "This limiting equation of state is obtained here assuming an electromagnetic interaction between the particles. We shall assume that it remains valid for the other possible interactions between particles, though there is at present no proof of this assumption." So, it is found for the gravity fields, whose source has isotropic pressure.

However, the source of gravitation is not pressure of the gravity field but the vacuum pressure, which balances impact of gravity on vacuum in the stationary space-time. In cosmology, the equation of state for a perfect fluid corresponds to equation of state for low gravitating sphere. Theoretical estimation of the deceleration parameter is confirmed by the observed accelerating expansion of the universe.

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