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Five-dimensional Anisotropic Bianchi Type-I Cosmological Models with Constant Deceleration Parameter

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Abstract

The present study deals with a spatially homogeneous and anisotropic five dimensional Bianchi type-I cosmological models filled with perfect fluid. The five dimensional Einstein's field equations have been solved by applying the variation law for generalized hubble's parameters which was first proposed by Berman (1983) in FRW models and gives constant value of deceleration parameter. Two different physically viable models ($n \neq 0$, $n = 0$) of the universe exhibits power law and exponential expansion. Also physical and kinematical properties of the models are discussed.

Keywords: Five-dimensional, Bianchi Type-I, Hubble's parameter, deceleration parameter, cosmological model.

1. Introduction

At the present state of evolution, the universe is spherically symmetric and matter distribution in it on the whole isotropic and homogeneous. But in its early stages of evolution it could not had such a smoothed out picture because near the big bang singularity neither the assumption of spherical symmetry nor of isotropy can be strictly valid. Anisotropy of the cosmic expansion, which is supposed to be damped out in the course of cosmic evolution, is an important quantity.

Recent experimental data indicate the existence of an anisotropic phase of the cosmic expansion that approaches an isotropic one. For simplification and description of the large scale structure and behavior of the actual universe anisotropic Bianchi type-I models have been considered by many authors [Reddy (1987), Reddy and Venkateswarlu (1987), Lorenz-Petzold (1989), Venkateswarlu and Reddy (1990), Singh and Agrawal (1991), Gujman (1993), Chimento wt. al. (1997), Bali and Gokhroo (2001), Bali and Keshav(2003)] have studied anisotropic Bianchi type-I models in different manner. Also Bali and Anjali (2004) have been studied an anisotropic Bianchi type-I magnetized bulk viscous fluid string dust cosmological model, Kilinc (2004) has considered an anisotropic Bianchi type-I universe with variable gravitational and cosmological constants in the presence of a perfect fluid, Saha (2005,2006a,2006b) and Pradhan and Pandey

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(2006) have investigated Bianchi-I models with anisotropic background to study the possible effects of anisotropy in the early universe on the present day observation

In recent years, there has been considerable interest in string cosmology. Many authors have obtained cosmic string in different Bianchi type space times [Pradhan et al. (2009), Amirhashci and Zainuddin (2010) and Tripathi et al. (2009, 2010)] also Pradhan A and Chouhan D S (2011) have studied anisotropic Bianchi type I models in string cosmology. The advantages of these anisotropic models are that they have a significant role in the description of the evolution of the early phase of the universe and they help in finding more general cosmological models. Saha and Visinescu (2010) and Saha et al.(2010) have studied Bianchi type-I models with cosmic string in presence of magnetic flux. The study of higher dimensional physics is an important because of several prominent results obtained in the development of the superstring and other field theory. Krori et al. (1994) constructed a Bianchi type-I string cosmological model in higher dimensional space time and obtained that matter and string coexist throughout the evolution of the universe. Several authors constructed five dimensional cosmological models in various aspects.

Thus in the present paper, we have obtained exact solutions of five dimensional anisotropic Bianchi type-I space-time in which the source of matter distribution is perfect fluid on the lines of Suresh Kumar and C. P. Singh (2007).

The paper is organized as follows: In Sec.2, we give a introduction about the field equations. In Sec.3, we deal with an exact solution of the field equations with perfect fluid. In section-4,5, we have discussed the cosmological model with physical behavior for both the models $(n \neq 0, n = 0)$ and in the last section-6, we summarize and conclude the results.

2. The metric and Einstein's field equations in V_5

We consider five dimensional spatially homogeneous and anisotropic Bianchi type-I metric

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)(dy^2 + dz^2) + C^2(t)du^2 \quad (1)$$

where A, B and C are functions cosmic time t alone.

The five dimensional Einstein field equations in case of perfect fluid is given by

$$R_{ij} - \frac{1}{2} g_{ij} R = 8\pi T_{ij}, \quad (i, j = 1,2,3,4,5) \quad (2)$$

The energy momentum tensor T_{ij} for perfect fluid distribution is taken as

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} \quad (3)$$

where p is pressure and ρ is the energy density for a fluid $u^i = (0,0,0,0,1)$ is the five velocity of the particle and $u^i u_i = -1$. R_{ij} is Ricci tensor, R is Ricci scalar.

In a co-moving coordinate system, the field equations (2) for the anisotropic Bianchi type-I space time (1) in case of (3) lead to the following set of independent differential equations:

$$\frac{2\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{2\dot{B}\dot{C}}{BC} + \frac{\dot{B}^2}{B^2} = 8\pi p, \tag{4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = 8\pi p, \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = 8\pi p, \tag{6}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{2\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}^2}{B^2} = 8\pi\rho. \tag{7}$$

Here dot indicate ordinary differentiation with respect to t.

The field equations (4)-(7) are four equations involving five unknown A, B, C, ρ and p.

The energy conservation equation $T^{ij}_{;j} = 0$, leads to the expression:

$$\dot{\rho} + (\rho + p)(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C}) - \frac{\dot{A}}{A} = 0. \tag{8}$$

3. Solutions of the Einstein's field equations in V_5

Einstein's field equations (4)-(7) are higher non-linear differential equation. The solutions of the field equations are solved by applying a variation law for generalized Hubble parameter proposed by Berman (1983) in space-time (1) which gives a constant deceleration parameters. Many authors have considered cosmological models with constant deceleration parameter. Reddy et al. (2006,07) have presented LRS Bianchi type-I models with constant deceleration parameter in scalar tensor and scale covariant theories of gravitation. Also Suresh Kumar and C. P. Singh (2006,07) have investigated LRS Bianchi type-II models with constant deceleration parameter in general relativity, Guth's inflationary theory and self creation theory of gravitation. Suresh Kumar and C. P. Singh (2007) have obtained a exact solutions for a spatially homogeneous and anisotropic Bianchi-type-I space-time with perfect fluid in general relativity by applying a special law of variation for Hubble's parameters that yield a constant value of deceleration parameter. Recently Jumale et. al. (2014) have obtained exact solutions for specially homogeneous and anisotropic five-dimensional Bianchi type-I cosmological model representing massive string.

Using the special law of variation for the Hubble parameter given by Berman (1983), gives a constant value of deceleration parameter. Hence, the law read as

$$H = Da^{-n} = D[AB^2C]^{\frac{-n}{4}} \tag{9}$$

where $D > 0$ and $n \geq 0$ are constant.

Hubble parameter in anisotropic model is given by

$$H = \frac{\dot{V}}{V} = \frac{1}{4}[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C}] \tag{10}$$

where dot denotes derivative with respect to *t*. Also

$$H = \frac{1}{4}(H_1 + H_2 + H_3 + H_4) \tag{11}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = H_3 = \frac{\dot{B}}{B}$, $H_4 = \frac{\dot{C}}{C}$ are directional hubble's parameter in the direction of *x*,*y*,*z* and *u* direction respectively.

From (9) and (10) we get,

$$\frac{1}{4}[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C}] = D[AB^2C]^{\frac{-n}{4}}. \tag{12}$$

Integration of (12) gives

$$AB^2C = (nDt + c_1)^{4/n}, \quad (n \neq 0) \tag{13}$$

$$AB^2C = c_2^4 e^{4Dt}, \quad (n = 0) \tag{14}$$

where c_1 and c_2 are constant of integration.

It is mention here that we have used $H = Da^{-n}$, $D > 0$, $n \geq 0$ to get the above equation. Thus we obtain two values of the average scale factor corresponding to two different models of the universe.

The value of deceleration parameter (q), is then found to be

$$q=-\frac{V\ddot{V}}{\dot{V}^2}, \tag{15}$$

Substituting (13) into (15), we get

$$q=(n-1) \tag{16}$$

which is constant value of deceleration parameter.

Subtracting (4) from (5) and taking integral of the resulting equation two times, we get

$$\frac{A}{B}=d_1\exp[x_1\int(AB^2C)^{-1}dt], \tag{17}$$

$$\frac{A}{C}=d_2\exp[x_2\int(AB^2C)^{-1}dt] \tag{18}$$

$$\frac{B}{C}=d_3\exp[x_3\int(AB^2C)^{-1}dt], \tag{19}$$

where d_1,d_2,d_3 and x_1,x_2,x_3 are constants of integration.

From equations (17)-(19), the metric functions can be written as

$$A=a_1(AB^2C)^{1/4}\exp[b_1\int(AB^2C)^{-1}dt], \tag{20}$$

$$B=a_2(AB^2C)^{1/4}\exp[b_2\int(AB^2C)^{-1}dt]], \tag{21}$$

$$C=a_3(AB^2C)^{1/4}\exp[b_3\int(AB^2C)^{-1}dt], \tag{22}$$

where

$$a_1=(d_1^2d_2)^{1/4},\quad a_2=(d_1^{-1}d_3)^{1/4},\quad a_3=(d_2^{-1}d_3^{-2})^{1/4}, \tag{23}$$

And

$$b_1=\frac{2x_1+x_2}{4},\quad b_2=\frac{x_3-x_1}{4},\quad b_3=-(\frac{x_2+2x_3}{4}).$$

Satisfy the following relation

$$a_1 a_2^2 a_3 = 1, \qquad b_1 + 2b_2 + b_3 = 0\,. \tag{24}$$

4. Five dimensional Cosmological Model of the Universe when $n \neq 0$

This model is based on the exact solutions of five dimensional Bianchi type-I space-time filled with perfect fluid.

Using (13) in (20-22), we obtain the metric functions as

$$A(t) = a_1 (nDt + c_1)^{1/n} \exp\left[\frac{b_1 (nDt + c_1)^{\frac{n-4}{n}}}{D(n-4)}\right], \tag{25}$$

$$B(t) = a_2 (nDt + c_1)^{1/n} \exp\left[\frac{b_2 (nDt + c_1)^{\frac{n-4}{n}}}{D(n-4)}\right], \tag{26}$$

$$C(t) = a_3 (nDt + c_1)^{1/n} \exp\left[\frac{b_3 (nDt + c_1)^{\frac{n-4}{n}}}{D(n-4)}\right]. \tag{27}$$

Putting (25-27) in (6) and (7), the pressure and energy density of the model is given by

$$8\pi p = -3D^2 (n-2)(nDt + c_1)^{-2} + (nDt + c_1)^{-\frac{8}{n}} (b_1^2 + 3b_2^2 + 2b_1 b_2)\,, \tag{28}$$

$$8\pi\rho = 6D^2 (nDt + c_1)^{-2} + (nDt + c_1)^{-\frac{8}{n}} (2b_1 b_2 + 2b_2 b_3 + b_1 b_3 + b_2^2)\,. \tag{29}$$

Now we find expression for some other cosmological parameters of the model. The anisotropy parameter A is define as

$$A = \frac{1}{4} \sum_{i=1}^{4} \left(\frac{H_i - H}{H} \right)^2 \tag{30}$$

The directional Hubble factors $H_i \, (i = 1,2,3,4)$ in the direction of x, y, z and u read as

$$H_i = D(nDt + c_1)^{-1} + b_i (nDt + c_1)^{-4/n}\,. \tag{31}$$

The expansion scalar θ is given by

$$\theta = 4H = 4D(nDt + c_1)^{-1}. \tag{32}$$

The anisotropy parameter A is define as

$$A = \frac{1}{4D^2}(b_1^2 + 2b_2^2 + b_3^2)(nDt + c_1)^{\frac{2n-8}{n}}. \tag{33}$$

The special volume are found to be

$$V = (nDt + c_1)^{1/n}. \tag{34}$$

$$\sigma^2 = \frac{1}{2}[(nDt + C_1)^{-8/n}(b_1^2 + 2b_2^2 + b_3^2)]. \tag{35}$$

From the above results, it can be seen that the spatial volume is zero at $t = -(c_1 / nD)$, and expansion scalar is infinite, which shows that the universe starts with zero volume with infinite rate of expansion. The scalar factors also vanishes at $t = -(c_1 / nD)$ and singularity obtained at initial stage. As t increases, the scale factors and spatial volume increases but expansion scalar decreases. Thus the rate of expansion slows down with increase in time. Also $\rho, p, H_1, H_2, H_3, H_4, A$ and σ^2 decrease at t increases. As $t \to \infty$, scale factors and volume become infinite whereas $\rho, p, H_1, H_2, H_3, H_4, \theta, A$ and σ^2 tend to zero. Therefore, the model would essentially give an empty universe for large time *t*. The ratio σ / θ tend to zero as $t \to \infty$ provided $n < 4$. Therefore, the model approaches isotropy for large values of *t*. Hence the model representing shearing, nonrotating and expanding model of the universe at late times.

5. Five dimensional Cosmological Model of the Universe when $n = 0$

This model is based on the exact solutions of five dimensional Bianchi type-I space-time filled with perfect fluid. Using (14) in (20-22), we obtain the metric functions as

$$A(t) = a_1 c_2 \exp[Dt - \frac{b_1}{4DC_2^4} e^{-4Dt}], \tag{36}$$

$$B(t) = a_2 c_2 \exp[Dt - \frac{b_2}{4DC_2^4} e^{-4Dt}], \tag{37}$$

$$C(t) = a_3 c_2 \exp[Dt - \frac{b_3}{4DC_2^4} e^{-4Dt}]. \tag{38}$$

Putting (25-27) in (6) and (7), the pressure and energy density of the model is given by

$$8\pi p = 6D^2 + (b_1^2 + 3b_2^2 + 2b_1 b_2) c_2^{-8} e^{-8Dt}, \tag{39}$$

$$8\pi\rho = 6D^2 + (2b_1 b_2 + 2b_2 b_3 + b_1 b_3 + b_2^2) c_2^{-8} e^{-8Dt}. \tag{40}$$

The solutions (25-27) satisfy the energy conservation equation (8) identically and hence represent exact solutions of the Einstein's field equation (5-9).

The directional Hubble factors $H_i (i = 1,2,3,4)$ in the direction of x, y, z and u read as

$$H_i = D + b_i c_2^{-4} e^{-4Dt}, \quad (i = 1,2,3,4) \tag{41}$$

The expansion scalar θ is given by

$$\theta = 4H = 4D. \tag{42}$$

The anisotropy parameter A is define as

$$A = \frac{1}{4D^2}(b_1^2 + 2b_2^2 + b_3^2) c_2^{-8} e^{-8Dt}. \tag{43}$$

The special volume (V) are found to be

$$V = c_2 e^{Dt}. \tag{44}$$

$$\sigma^2 = \frac{1}{2}[(b_1^2 + 2b_2^2 + b_3^2) c_2^{-8} e^{-8Dt}]. \tag{45}$$

From the above results, it can be seen that the spatial volume, scale factors, pressure, energy density and cosmological parameters are constant at $t = 0$. Thus universe starts evolving with a constant volume and expands with exponential. As t increases, the scale factors and spatial volume increases exponentially while the pressure, energy density, anisotropy parameter and shear scalar decrease. It is to noted that the expansion scalar is constant throughout the evolution of universe and therefore the universe exhibits uniform exponential expansion in this model.

As $t \to \infty$, the scale factors and volume of the universe become infinitely large whereas the scalar field, anisotropy parameter and shear scalar tend to zero. The pressure, energy density and Hubble's factors become constants such that $p = -\rho$. The model approaches isotropy for large time *t*. Thus the model representing shearing, nonrotating and expanding model of the universe with a finite start approaching to isotropy at late times. For $n = 0$, $q = -1$, this value of deceleration parameter leads to $dH/dt = 0$, gives the greatest value of Hubble's parameters and the fastest rate of expansion of the universe. Therefore the solutions presented in this model are consistent with the observations.

6. Concluding Remark

In the present paper, we have obtained a spatially homogeneous and anisotropy Bianchi type-I space-time with perfect fluid. These field equations have been solved using a special law of variation of Hubble's parameter that yields a constant value of deceleration parameter. Two models have been obtained $(n \neq 0, n = 0)$ also some important cosmological parameters have been obtained for both the models and physical behavior of the models is discussed in detail.

For $n \neq 0$, scale factors and volume vanishes. As $t \to \infty$, the pressure, energy density become negligible whereas the scale factors and spatial volume become infinitely large.

For $n = 0$, density being finite. The universe exhibit exponential expansion and expand uniformly. Both models represent shearing, non-rotating and expanding universe, which approaches to isotropy for large values of t.

The solutions obtained in the models are consistent with recent observations. The solutions presented in the paper are new and may useful in the analysis of evolution of universe in anisotropic Bianchi type-I space-time in general relativity.

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