Article

Aharonov-Bohm Effect with a Chiral Gauge Transformation

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Abstract

In this work we study the Aharonov-Bohm (AB) effect in a Born-Fedorov chiral media, being this chiral formalism a nonlocal approach like the quantum AB effect. Our result shows that the nonlocal chiral parameter T acts as a hidden factor.

Keywords: Chiral gauge transformation, Aharonov-Bohm effect.

1. Introduction

The Aharonov–Bohm (AB) effect, in which a charged particle is affected by a magnetic field even as it travels through a region in which the magnetic field is zero, has been experimentally confirmed using an electron in a superconducting ring and other materials. Even in a non-localized magnetic field, the wave function of a charged particle obtains a phase that is proportional to the area of a closed loop. This effect is also referred to as the nonlocal AB effect; dynamical nonlocality arises from the structure of the equations of motion. Dynamical non-locality was first introduced by Aharonov et al [1], in order to explain the nonlocality of topological phenomena such as the AB effect because it conclusively proved that a magnetic (or electric) field inside a confined region can have a measureable impact on a charged particle which never travelled inside the region.

In order to represent the closest correspondence between measurement and theory, Aharonov introduced nonlocal interactions between the particle and field. This was in contrast to the prevailing approach of reifying local interactions with (unphysical) non-gauge invariant quantities outside the confined region, such as the vector (and/or scalar) potential. Dynamic non-locality is generic and can be found in almost every type of quantum phenomenon [1-4]. A good example of dynamical nonlocality is the two-slit experiment, the quintessential example of the dual character of quantum mechanics. The initial incoming particle seems to behave as a wave when falling on the (left and right) slits, but when recorded on the screen, its wavefunction "collapses" into that of a localized particle. Here we study the AB effect in a chiral media which

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has nonlocal constitutive relations (Born-Fedorov approach [4]), and we show that the Uckun's approach is not acceptable to obtain the AB effect.

2. Nonlocal electromagnetic potential and gauge symmetry

The Maxwell equations for the macroscopic free electromagnetic fields (without charge and current), are well known, and we often write them in terms of electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$:

$$\nabla\times\mathbf{E}=-\frac{\partial}{\partial t}\mathbf{B}\,,\qquad \nabla\cdot\mathbf{B}=0\,,\qquad \nabla\times\mathbf{H}=\frac{\partial}{\partial t}\mathbf{D}+\mathbf{J}\,,\qquad \nabla\cdot\mathbf{D}=\rho\,. \tag{1}$$

These equations, however, are not complete because six more equations, the constitutive relations, have to be added relating the electric field $\mathbf{E}$, the magnetic induction $\mathbf{B}$, the displacement field $\mathbf{D}$ and the magnetic field $\mathbf{H}$ to each other. These constitutive relations are completely independent of the Maxwell equations which involve only the fields and their sources. The constitutive relations are concerned with the equations of motion of the constituents of the medium in an electromagnetic field [5-7].

However, the second and third of these equations can be reduced to mathematical identities if we work in terms of the scalar and vector potentials V and $\boldsymbol{A}$, defined by with the nonlocality definitions of Born-Fedorov [4, 8, 9]:

$$\mathbf{B}=\mu_T\left[\mathbf{H}+T(\nabla\times\mathbf{H})\right],\qquad\qquad \mathbf{D}=\epsilon_T\left[\mathbf{E}+T(\nabla\times\mathbf{E})\right], \tag{2}$$

$$\mathbf{B}=\nabla\times\left(\mathbf{A}+T\nabla\times\mathbf{A}\right)=\nabla\times\mathbf{F}\,,\ \left(\mathbf{A}+T\nabla\times\mathbf{A}\right)=\mathbf{F}\,,\ \nabla\cdot\mathbf{F}=\nabla\cdot\mathbf{A}\,,\mathbf{E}=-\frac{\partial}{\partial t}\mathbf{F}-\nabla V\,. \tag{3}$$

The nonlocal character of equations (2) needs to be noticed, because the magnetization of media depends not only on H, but also on the circulation of H, (rotH). Here, we have nonlocal constitutive relations that we need to study the nonlocal AB effect.

On the other hand the Born-Fedorov formulation for chiral media is the only one that can be work at zero frequency. When we compare with other formulations like the Uckun's work [10], where he mixes a local constitutive relation of Engetha formulation [7] with a nonlocal one of Born [11], we can see that this approach is not auto consistent, so it is not possible to study the nonlocal AB effect. For this reason Uckun paper must be corrected using the complete formalism of Born-Fedorov [8] by considering a factor $1-(k_0T)^2$ where T is the nonlocal chiral parameter.

The inconsistency of Uckun's article was first detected in [12] and the AB effect cannot be studied with this approach, because at zero frequency, the chiral Uckun's admittance goes to zero. In [13] we review the truly relationships between $\mathbf{E}$, $\boldsymbol{B}$, $\mathbf{D}$ and $\mathbf{H}$ in chiral media.

Maxwell's equations (1), can then be reduced to two equations for the potentials,

$$\nabla^2 V + \frac{\partial}{\partial t}\nabla\cdot\mathbf{F} = -\frac{\rho}{\varepsilon_0}, \qquad \nabla\times\nabla\times\mathbf{F}(1-(k_0T)^2) + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{F} = \mu_0 J - \frac{1}{c^2}\frac{\partial\nabla V}{\partial t} \quad , \tag{4}$$

which can often simplify calculations [4]. We might therefore be tempted to immediately abandon the six quantities that define the electromagnetic field in favour of using the four quantities that determine the potentials. However, this is not so straightforward as V and $\boldsymbol{A}$ are not uniquely determined. The $\boldsymbol{E}$ and $\boldsymbol{B}$ fields stay the same after the following transformation,

$$\mathbf{F}\to\mathbf{F}+\nabla\chi \ , \qquad V\to V-\frac{\partial}{\partial t}\chi\,, \tag{5}$$

where χ is an arbitrary function of space and time; the mapping (5) is known as a *gauge transformation*. Classically we don't ascribe any physical meaning to the potentials precisely because of this symmetry you can't measure something that isn't uniquely defined.

3. Non local AB effect

Let's now consider a specific situation. A line of coils carrying electric current is placed along the z axis, as shown in the figure below. The setting for the AB effect is very similar to the two-slit experiment, with just one difference: immediately beyond the two slits and in between them is a very fine and long solenoid, ideally infinitely long, producing a magnetic field that is confined entirely within the tube of the solenoid. When the current in the solenoid is switched on, or in other words, when there is a magnetic field **B** present inside the solenoid, the phase of the electrons changes and their interference pattern is shifted. The configuration of the AB effect in a chiral media is depicted in figure 1:

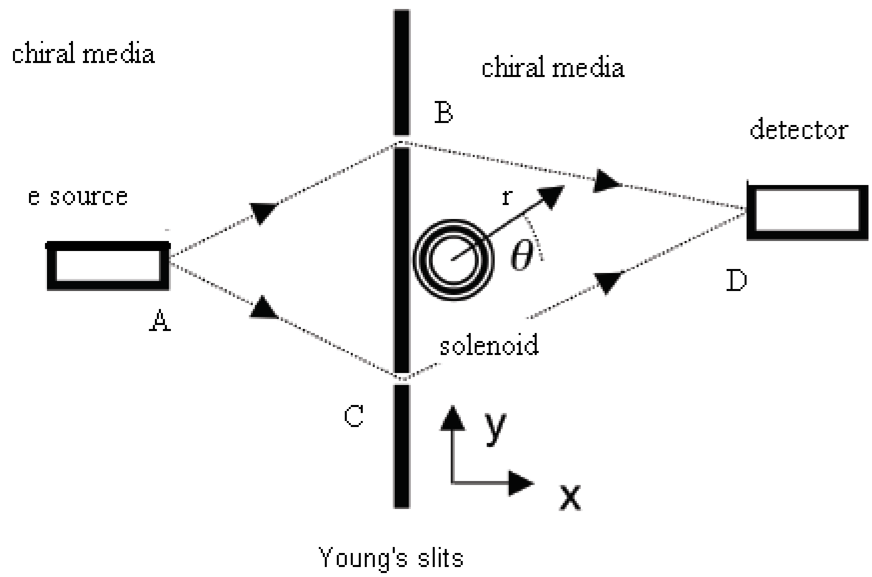

Figure 1. The Aharanov–Bohm experiment immersed in a Born-Fedorov media

Here we set $\mathbf{B}=\mu_T\left[\mathbf{H}+T(\nabla\times\mathbf{H})\right]=0$ for any value of the nonlocal chiral parameter T, that is, $\mathbf{B}=\nabla\times\left(\mathbf{A}+T\nabla\times\mathbf{A}\right)=\nabla\times\mathbf{F}=0$ outside the solenoid. So $\mathbf{F}$ is derived from a scalar $\left(\mathbf{A}+T\nabla\times\mathbf{A}\right)=\mathbf{F}=\nabla(g)$. Here it is necessary to determine the scalar g in this configuration.

Now we will determine the vector potential that describes this situation. Equations (4) at zero frequency, $\omega=0$, in the Coulomb gauge, where V=0, χ is chosen such that $\nabla\cdot\mathbf{F}=\nabla\cdot\mathbf{A}$, so from (4) we have $\nabla^2\mathbf{F}=-\mu_0\mathbf{J}$ which is just a vector version of Poisson's equation for $a/r>1$, where a is the radius of the coils. Solving for $\mathbf{F}$ we have:

$$\mathbf{F}=\frac{\mu_0}{4\pi}\int d^3r'J(\mathbf{r}')/\left|\mathbf{r}-\mathbf{r}'\right|, \tag{6}$$

and the current density is given by:

$$\mathbf{J(r)}=nI\delta\mathbf{(r\text{-}a)(}\cos(\theta)\mathbf{y}-\sin(\theta)\mathbf{x}), \tag{7}$$

where there are n turns of the coil (radius a) per unit length along the z axis, with a current I flowing through each. Inserting (7) into (6) and expanding the resulting expression to leading order in a/r (the first order term integrates to zero), one obtains,

$$\mathbf{F}=\frac{\mu_0 nIa^2}{4\pi r^2}\int_{-\infty}^{\infty}dz'\int_0^{2\pi}d\theta'\cos(\theta')\mathbf{y}(1+z'^2/r^2)^{-3/2}, \qquad \mathbf{F}=\frac{\mu_0 nIa^2}{4r}\mathbf{y}\int_{-\infty}^{\infty}d\zeta(1+\zeta^2)^{-3/2},$$

where we have a point of observation at $\theta = 0$, $z = 0$, and assumed that the line of coils has an infinite length. The final integral can be performed, with the result $\int_{-\infty}^{\infty} d\zeta (1+\zeta^2)^{-3/2} = 2$, so that:

$$\mathbf{F}(r,0,0) = \frac{\mu_0 n I a^2}{2r} \mathbf{y} \rightarrow \mathbf{F} = \mathbf{A} + T\nabla \times \mathbf{A} = \frac{\Phi\theta}{2\pi r}, \tag{8}$$

where the symmetry of the situation determines the direction of the vector in the second step, and $\Phi = \mu_0 n I \pi a^2$. Equation (8) is the vector potential that describes the field of a long, infinitesimally thin line of coils. However, (8) can also be re-written as the gradient of a scalar function $\mathbf{F}(r) = \nabla(\frac{\Phi\theta}{2\pi})$. This is a pure gauge field with $\mathbf{F}(r) = \nabla g$, such that $g = \Phi\theta / 2\pi = \Phi(n + \theta / 2\pi)$ in the system of cylindrical coordinates $\mathbf{r} = (r, \theta, z)$ and n is the integral winding number. If we use Stokes' theorem at this point the shift is given by the relation:

$$\delta = \zeta_1 - \zeta_2 = \frac{q}{\hbar}(0) + \frac{q}{\hbar}(\Phi)(1 - \frac{2T}{a}); \tag{9}$$

when the current in the solenoid is switched on, or in other words, when there is a magnetic field **B** presents inside the solenoid, the phase of the electrons changes and their interference pattern is shifted by δ.

4. Conclusion

In this work we have studied the Aharonov-Bohm effect in a Born-Fedorov chiral media, being this chiral formalism a nonlocal approach like the AB effect. Our result show that the nonlocal chiral parameter T acts as a hidden factor. We show that the Uckun's approach is not self-consistent because we cannot obtain the AB effect with this approach.

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