Article

On the Physical Degree of Freedom Count in Terms of Lagrangian parameters

V. Barrera-Figueroa[1], A. Iturri-Hinojosa[2] & J. López-Bonilla[*2]

[1]Posgrado en Tecnología Avanzada, SEPI-UPIITA, IPN, Av. IPN 2580, CP 07340, CDMX, México
[2]ESIME-Zacatenco, IPN, Edif. 5, 1er. Piso, Lindavista 07738, CDMX, México,

Abstract

We give a simple motivation of the Díaz-Higuita-Montesinos formula to obtain the number of physical degrees of freedom in singular systems.

Keywords: Constrained Hamiltonian systems, singular Lagrangians.

1. Introduction

Recently in [1] was obtained the following expression to determine the number of physical degrees of freedom (NPDF) for systems governed by singular Lagrangians:

$$NPDF = N - \frac{1}{2}(l + g + e), \tag{1}$$

where only appear quantities from the Lagrangian scheme, in fact, N, e, l, and g are the total number of generalized coordinates $q_j(t)$, effective gauge parameters [1], genuine constraints and gauge identities [2-5], respectively. This same calculation can be realized via the Hamiltonian formula [6]:

$$NPDF = N - N_1 - \frac{1}{2}N_2 \, , \tag{2}$$

using only concepts from the Rosenfeld-Dirac-Bergmann approach [7-15], where N_1 and N_2 are the total number of first-and second-class constraints [16-18], respectively; let's remember that N_2 is an even number [11, 19] and that the number of degrees of freedom is the same for Hamiltonian and Lagrangian formalisms [20].

In Sec. 2 we exhibit a simple manner to motivate the relation (1).

* Correspondence: J. López-Bonilla, ESIME-Zacatenco-IPN, Edif. 5, Col. Lindavista CP 07738, CDMX, México
E-mail: jlopezb@ipn.mx

2. Díaz-Higuita-Montesinos expression

In [20] was established the formula (written in our present notation):

$$l = N_1 + N_2 - N_1^{(p)} \, , \tag{3}$$

being $N_1^{(p)}$ the total number of first-class primary constraints. On the other hand, the total Hamiltonian H_T already contains all the information about the gauge freedom of the theory, through the first-class primary constraints included in it [20], that is, H_T is the first-class Hamiltonian plus arbitrary linear combinations of the first-class primary constraints $\psi_a = V_{(a)}^m \varphi_m$, $m = 1, \dots, M$, where M is the amount of primary constraints, such that [12, 16]:

$$C_{(N_1+N_2)xM} \vec{V}_{(a)} = 0, \quad a = 1, \dots, N_1^{(p)}, \quad C = \left(\{\varphi_j, \varphi_m\}\right), \quad j = 1, \dots, N_1 + N_2, \tag{4}$$

hence $H_T = H_{first-class} + v^a \psi_a$ with $v^a(t)$ totally arbitrary, but we may remember that in the Lagrangian approach [2, 3, 5] the number of arbitrary functions into the gauge transformations coincides with the number of gauge identities, therefore:

$$g = N_1^{(p)} = M - rank\ C. \tag{5}$$

If we accept that the Dirac's conjecture [2, 12, 13, 16, 20-24] is valid, then the N_1 first-class constraints (primary and secondary) generate gauge symmetries into the Hamiltonian formalism and it is natural to identify this freedom with the number of effective gauge parameters in the Lagrangian process:

$$e = N_1 \, , \tag{6}$$

thus, from (3), (5) & (6) we have that $N_1 = e$, $N_2 = l + g - e$ and $N_1^{(p)} = g$, therefore (2) implies the formula (1) obtained by Díaz-Higuita-Montesinos [1].

In [6] was showed how to apply (1) to several Lagrangians studied in [2, 25-27].

Received June 14, 2016; Accepted June 26, 2016

References

1. B. Díaz, D. Higuita, M. Montesinos, *Lagrangian approach to the physical degree of freedom count,* J. Math. Phys. **55** (2014) 122901
2. H. J. Rothe, K. D. Rothe, *Classical and quantum dynamics of constrained Hamiltonian systems*, World Scientific Lecture Notes in Physics **81**, Singapore (2010)
3. P. Lam, J. López-Bonilla, R. López-Vázquez, G. Ovando, *Lagrangians: Symmetries, gauge identities, and first integrals*, The SciTech, J. of Sci. & Tech. **3**, No. 1 (2014) 54-66
4. P. Lam, J. López-Bonilla, R. López-Vázquez, G. Ovando, *On the gauge identities and genuine constraints of certain Lagrangians,* Prespacetime Journal **6**, No. 3 (2015) 238-246
5. P. Lam, J. López-Bonilla, R. López-Vázquez, G. Ovando, *Matrix method to construct point symmetries of Lagrangians*, Bull. of Kerala Mathematics Assoc. **12**, No. 1 (2015) 43-52
6. P. Lam, J. López-Bonilla, R. López-Vázquez, M. Maldonado-Ramírez, *On a recent formula to determine the physical degree of freedom count,* Prespacetime Journal **7**, No. 9 (2016) 1306-1312
7. L. Rosenfeld, *On the quantization of wave fields*, Ann. der Phys. **5** (1930) 113-152
8. P. Hanson, T. Regge, C. Teitelboim, *Constrained Hamiltonian systems,* Accad. Naz. dei Lincei, Rome (1976)
9. C. A. Hurst, *Dirac's theory of constraints,* in 'Proc. Recent developments in Mathematical Physics', Eds. H. Mitter, L. Pitter; Springer-Verlag, Berlin (1987) 18-52
10. P. Bergmann, *The canonical formulation of general-relativistic theories: The early years, 1930-1959,* in 'Einstein and the history of general relativity', Eds. D. Howard, J. Stachel; Birkhäuser, Boston (1989) 293-299
11. A. Wipf, *Hamilton's formalism for systems with constraints,* in 'Canonical gravity: From classical to quantum', Eds. J. Ehlers, H. Friedrich; Springer-Verlag, Berlin (1994) 22-58
12. M. Henneaux, C. Teitelboim, *Quantization of gauge systems,* Princeton University Press, NJ (1994)
13. J. Earman, *Tracking down gauge: an ode to the constrained Hamiltonian formalism,* in 'Symmetries in Physics. Philosophical Reflections', Eds. Katherine Brading, Elena Castellani; Cambridge University Press (2003) 140-162
14. D. C. Salisbury, *Rosenfeld, Bergmann, Dirac and the invention of constrained Hamiltonian dynamics,* arXiv: physics/0701299v1 [physics.hist-ph] 25 Jan 2007
15. D. C. Salisbury, *Peter Bergmann and the invention of constrained Hamiltonian dynamics,* in 'Einstein and the changing worldviews of Physics', Eds. C. Lehner, J. Renn, M. Schemmel; Einstein Studies **12**, Birkhäuser, Boston (2012) 247-257
16. P. Dirac, *Lecture on quantum mechanics,* Yeshiva University, New York (1964)
17. M. J. Gotay, J. M. Nester, G. Hinds, *Presympletic manifolds and the Dirac-Bergmann theory of constraints,* J. Math. Phys. **19**, No. 11 (1978) 2388-2399
18. E. Castellani, *Dirac on gauges and constraints,* Int. J. Theor. Phys. **43**, No. 6 (2004) 1503-1514
19. J. N. Goldberg, *Second-class constraints,* in 'On Einstein's path. Essays in honor of Engelbert Schucking', Ed. A. Harvey; Springer-Verlag, New York (1999) 251-256
20. G. R. Allcock, *The intrinsic properties of rank and nullity of the Lagrange bracket in the one dimensional calculus of variations,* Phil. Trans. Roy. Soc. London **279**, No. 1290 (1975) 487-545
21. R. Cawley, *Determination of the Hamiltonian in the presence of constraints,* Phys. Rev. Lett. **42**, No. 7 (1979) 413-416
22. X. Gracia, J. M. Pons, *Gauge generators, Dirac's conjecture, and degrees of freedom for constrained systems,* Ann. of Phys. **187**, No. 2 (1988) 355-368
23. M. E. V. Costa, H. O. Girotti, T. J. M. Simes, *Dynamics of gauge systems and Dirac's conjecture,* Phys. Rev. D**32**, No. 2 (1985) 405-410
24. A. Cabo, D. Louis-Martínez, *On Dirac's conjecture for Hamiltonian systems with first-and second-class constraints,* Phys. Rev. D**42**, No. 8 (1990) 2726-2735

25. M. Henneaux, C. Teitelboim, J. Zanelli, *Gauge invariance and degree of freedom count*, Nucl. Phys. B**332**, No. 1 (1990) 169-188
26. M. Havelková, *Symmetries of a dynamical system represented by singular Lagrangians*, Comm. in Maths. **20**, No. 1 82012) 23-32
27. G. F. Torres del Castillo, *Point symmetries of the Euler-Lagrange equations*, Rev. Mex. Fís. **60** (2014) 129-135