Exploration

Charting Trends in the Mandelbrot Set & Showing Their Significance for Cosmology

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Abstract

The author has been exploring the Mandelbrot Set in unique ways, for about 30 years, by highlighting areas of the Complex plane where the magnitude of the iterand follows a trend. He discovered the Butterfly figure which appears when one colors in areas where the iterand's magnitude diminishes monotonically over three iterations, revealing patterns in $\mathcal{M}$ that may have significance for theories of Cosmology. The progression of form along either edge from the cusp has an evolutive trend that appears to elucidate the fundamental forces. He published a letter in *Amygdala* [1], and had conversations with Benoit Mandelbrot, in the late 1980s. But further advances in Mathematics, since that time, establish the importance of the Mandelbrot Set and affirm the relevance of this line of study to Physics. However; a rigorous presentation of these ideas to the Mathematics and Physics community has not been made, so the material presented here has had limited attention thus far, and much of the territory the author has mapped remains largely unexplored by others. This paper is the first in a series to present this body of research to the larger community.

Keywords: Mandelbrot Set, Julia Sets, cosmology, complex analysis, theoretical Physics.

Introduction

The Mandelbrot Set is perhaps the most complex object in Mathematics, and it is undoubtedly one of the most fascinating and rewarding mathematical objects to explore, but it is more than merely a colorful diversion, or a tapestry of wonderful filigrees and spirals to delight the eye and mind. It is, in fact, a catalog or map describing the complex dynamics for a simple quadratic formula, $z \to z^2 + c$, over the parameter space of the complex plane. Its significance to Physics is hard to overstate, however, because it catalogs a broad spectrum of dynamical systems and vibratory dynamics, corresponding to physical processes from the temperature at the Planck scale down to absolute zero, according to my model. This is possible because its scope is not limited to 2-dimensional systems. Trends found within $\mathcal{M}$, for the case of Complex numbers, $\mathbb{C}$, also arise in the Quaternions and Octonions, $\mathbb{H}$ and $\mathbb{O}$ – with 4 and 8 dimensions respectively – because the Mandelbrot Set is defined there too. The relevance of complex and hyper-complex numbers to Physics is emphasized in recent work by Michael Atiyah [2], and works by John Baez and John Huerta [3], and they figure prominently into the work of Geoffrey Dixon [4],

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Richard Lockyer [5], Corinne Manogue and Tevian Dray [6], and Frank D. 'Tony' Smith [7]. A prior *Prespacetime* article by this author, with Ray B. Munroe [8], also stresses their importance. While the Reals are scalars – constant, fixed, or static quantities – the other number types, $\mathbb{C}$, $\mathbb{H}$, and $\mathbb{O}$, display or embody increasing levels of dynamism – because of their imaginary dimensions – and this property becomes apparent under the iteration of functions within their domain.

The author was fascinated and mesmerized, when reading about the Mandelbrot Set (or $\mathcal{M}$) in *Scientific American*, in August of 1985 [9], and seeing for the first time images of this remarkable figure. A.K. Dewdney gave enough information about the simple sequence of calculations and program loops which could generate the wonderful complexity of $\mathcal{M}$ for me to write a program and create images on our IBM PC. But friends with better programming skills and tools allowed us to advance the cause more quickly, devising better and quicker ways to get the images of $\mathcal{M}$ to our computer screens. So with ideas on how to make the algorithm go quicker; I visited a friend, Mark Little, who could turn those ideas into code rather swiftly. Looking for a calculational shortcut; I reasoned that since some infinite series whose terms diminish in magnitude over successive values converge, this might work for the Mandelbrot Set formula as well. So Mark altered the code to save prior values of the variable *size* – the iterand magnitude – and then we put in a conditional which tested for the condition: current $size$ < last $size$ < previous $size$. Our program called that a point in $\mathcal{M}$, and went on to the next location/pixel, however what we actually saw was not what we expected, as the periphery was full of holes – like Swiss cheese – (as in figure 1 below) instead of the Set's familiar warty outline. The author's reaction was that it looked like a silhouette depicting the Cosmological Eras, which he had just learned about in Cosmology class at College, so we decided to investigate further.

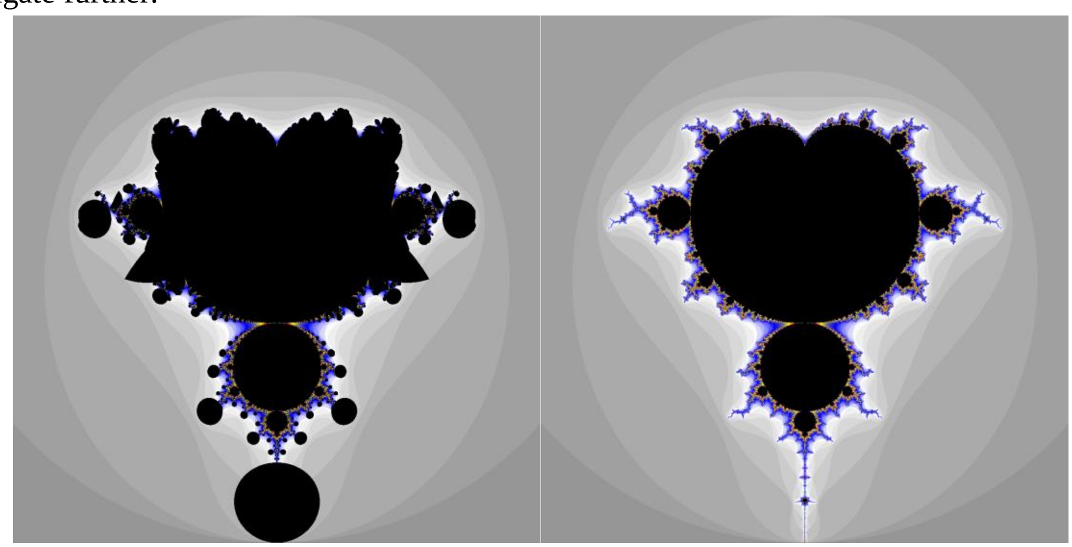

Figure 1. The periphery of the figure on the left resembles Swiss cheese, when compared to the familiar outline of the Mandelbrot Set shown on the right.

Since we knew this formula was not a true shortcut; we decided to color in those pixels meeting the condition of diminishing iterand magnitude, instead of painting them black, with a pixel's color determined by the number of iterations attained. This let us discern the shape of the figure meeting our test condition, a monotonic decline over three iterations. What we saw then is something I call the Mandelbrot Butterfly – depicted in figure 2 below. In this rendering; the green wings and discs show regions where the iterand comes to be monotonically diminishing on an even-numbered iteration, and the blue ones depict areas that resolve instead on odd-numbered iterations – so it is easy to see the distribution of odd and even sectors within the figure. This reveals a new level of detail and complexity, not seen when charting $\mathcal{M}$ using the conventional algorithm. The Mandelbrot Butterfly is not a separate object, though, but rather it depicts a fundamental behavior of the Mandelbrot Set that is seldom observed – because the standard algorithms do not reveal it. It significantly unveils patterns in the dynamism, highlighting the behavior of the generating function near the Misiurewicz points – places where the function infinitely repeats only after a certain number of iterations (the pre-period) have expired. The disks and plumes that populate the periphery of the Mandelbrot Butterfly reside behind and point to Misiurewicz points. So it is seen that the Butterfly figure and its family of related figures catalogues the pre-periodic behavior of $z \rightarrow z^2 + c$ under iteration, revealing something deep – hidden within the structure of $\mathcal{M}$ – while providing a whole new universe of form to observe and explore.

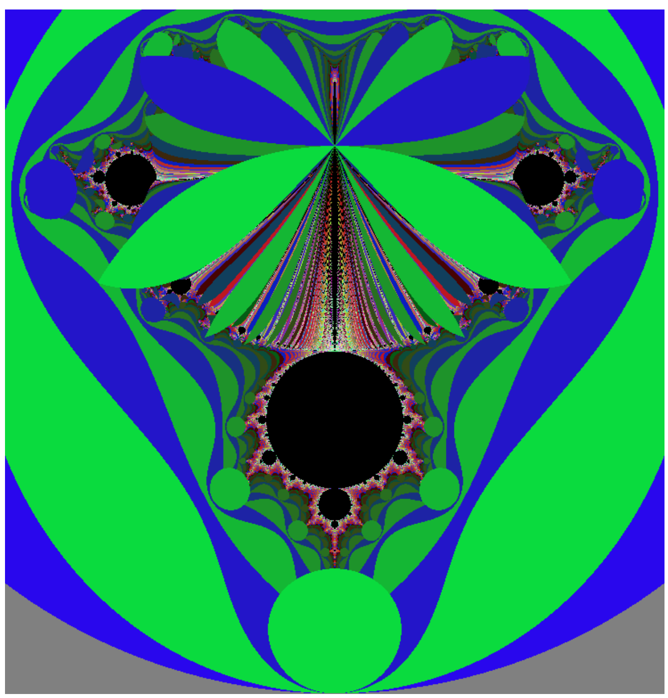

Figure 2. The Mandelbrot Butterfly

The remainder of the paper further describes what the author has learned about trends within $\mathcal{M}$, and in the form at its periphery, explaining how this object may have specific relevance to

Theoretical Physics and Cosmology, and showing why it *must* have relevance to Physics in general. There is already some application of the ideas to be discussed, though the author's discoveries are not widely known. The use of Julia Sets in Complex Analysis is widespread, and the Mandelbrot Set is a catalog for a family of Julias which possess a form that is self-similar with their point of origin within $\mathcal{M}$. This correspondence is most prominent or obvious at the Misiurewicz points, as first noted by Tan Lei [10], making them pivotal to our research into the Mandelbrot Set and Cosmology. These points include the centers of the spirals and multi-armed figures that inhabit the periphery of $\mathcal{M}$, and the tips of their projections.

As Robert L. Devaney describes [11]; locations on the periphery of $\mathcal{M}$ can be 'read' from the form we find at these points, as the number of 'spokes' at a branch point tell us the period of the 'bulb' to which it is attached, and so on. It is sad that the results presented in this paper are not more generally known in the Mathematics community, though; because they are obviously significant. Until now; the focus has been mainly on the relevance of this work to Physics, however, laying the groundwork for future development of physical theories derived from $\mathcal{M}$, without attempts to reach out to the Math or Fractal communities. More recently, though; a broader and deliberate outreach has begun, because it is apparent that recent developments in Math provide a springboard to progress, with many new resources to cultivate, where discoveries made over a quarter century ago now have importance for a larger range of fields than was previously imagined.

The Mandelbrot Mapping Conjecture

The author's observation, when the silhouette of the Butterfly figure was first seen, was that there is an analogy in the progression of forms begat by $\mathcal{M}$ – as they change in character when moving along either edge from the cusp to the tail – with the progression of the cosmological eras. This insight was likely precipitated by his taking courses in Astrophysics and Cosmology – which focused on the Big Bang model [12] and the recently proposed Inflationary universe scenario (I graduated in 1984 and Guth's lecture was in 1980). So of course he exclaimed "Oh my – it's the Big Bang" or something similar. He did not, however, fully appreciate the implications of this idea – not until much later.

It should be understood that to prove or disprove such a claim; one must undertake the study of the whole of theoretical Physics, with a special emphasis on Cosmology – because the Mandelbrot Set is seen to describe the entire evolution of the Cosmos and the full range of forces which bring the universe into being. Of course; this task does not need to be undertaken as a single act, but testing the thesis can instead be broken up into probing various specific correspondences, and testable claims or related topics of study, which can be investigated independently. Accordingly; the author has researched and elucidated elements of this program, and proven the soundness of various portions of his reasoning, while being less specific about the nature of that larger program. However; it became apparent recently that a concise statement is needed, of the entire research program that was inspired by my discoveries in the '80s.

Late in 2013 I framed a formal description, the Mandelbrot Mapping Conjecture or MMC, which can be stated thusly. "There exists sufficient dynamical variety in the Mandelbrot Set such that any physical system or process has a correspondence with a specific location in $\mathcal{M}$." My caveat is that we are talking about the Physics characterizing a process, or a local neighborhood in space at a certain time, and not all of the observables and qualia associated with that specific place and time. Studying this correspondence is expected to yield insights into the larger theoretical background in which the laws of Physics emerge, and into the means by which the laws of nature came to be what they are now. Perhaps more importantly; since $\mathcal{M}$ necessarily describes the whole evolution of the universe over cosmic time, it gives insights into the stage of cosmic evolution we now inhabit, in context with the larger scope of universal development. Intriguingly; the Cosmological framework based on the MMC supports both a cold dark end and a cyclical model equally well, depending on the exact details of one's interpretation, and the way in which $\mathcal{M}$ is depicted (see fig. 3 below). Further; noting that patterns and trends within $\mathcal{M}$ in $\mathbb{C}$ extend to the higher-dimensional cases $\mathbb{H}$ and $\mathbb{O}$ (4-dimensional and 8-dimensional algebras), this model allows for a range of Multiverse scenarios, though it constrains their number and kind. One might see the mini- $\mathcal{M}$ s around the periphery as a whole broad collection of universes, over a wide range of scale, but we leave that possibility to the reader's imagination for now.

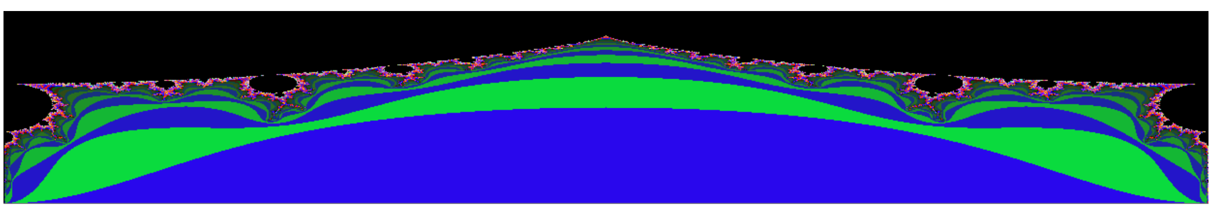

Figure 3. The Mandelbrot Set in a flat projection (where the origin is up) shows how a ball placed at the cusp would roll down the incline into an opening.

In the rendering of $\mathcal{M}$ above, it is laid flat; a range of concentric circles centered at the origin $(0, 0i)$ is displayed as rows of pixels – from left to right across the screen – so that distance of structures from the origin is shown by how far they are from the top of the figure. It is easy to see that a ball placed at the top would roll down from the cusp, and depending on its size might fall into one of the open spaces or bulbs around the periphery of $\mathcal{M}$. The cusp at $(.25, 0i)$ is the place with the highest potential energy in the Set, and it spontaneously breaks the symmetry such that lower potential wells become more and more likely trajectories – over time. But while a small enough ball is likely to fall into a well further up; a larger ball has no choice but to roll all the way down the hill. Our analogy depends upon $\mathcal{M}$'s boundary having a solid surface, upon which a ball might roll, with a gravitational pull toward the base of the figure, but this turns out to be an apt comparison. In the calculus of variations, one of the most well-known examples is the brachistochrone problem – finding the curve of most rapid descent, given such a setup.

The solution to this problem is a cycloid, which also turns out to be the bounding curve of the Mandelbrot Set's main body – as rendered as in figure 3. So we see that $\mathcal{M}$ offers an optimally time-efficient slope for balls to roll down, with a holey surface that traps some of the smaller balls before they reach the bottom, and sorts the captured balls by size. In the above representation; it is obvious that the cusp at $(.25, 0i)$ represents the energy peak, and that every other point on the periphery is at a lower level.

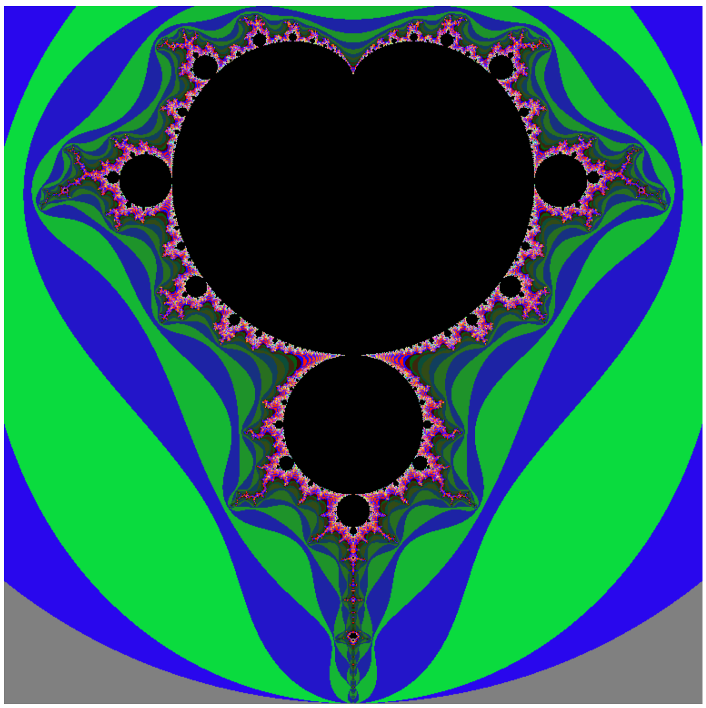

Figure 4. The Mandelbrot Set, with Real axis vertical, displays mirror symmetry about that axis, but is asymmetrical along it.

We also find that the cusp at $(.25, 0i)$ requires the most computation power to resolve, of any location in $\mathcal{M}$ – which makes it a calculational hot spot as well. It is known that the cusp is the point on the boundary where the repeller sets (places almost in $\mathcal{M}$) nearby require the highest number of iterations to calculate, or have the greatest density of high-iteration points, while the point $(-2, 0i)$ (in the lower-left and lower-right corners of Figure 3) is the location where the repeller sets surrounding it need the fewest iterations to resolve. The point $(-2, 0i)$, the base of the figure in other images, is a pre-periodic infinite repeating point, which arrives at its final repeated value after only one iteration! Thus it satisfies the relation $(c^2 + c)^2 + c = c^2 + c$, and is a Misiurewicz point. This location is referred to as the tip of the main antenna by Peitgen and Richter [13], who state that in general Misiurewicz points lie at the tips or branching points of the antennae on the 'bulbs' along the periphery. In figure 3; it is seen that the repeller sets (bands of color) slope away from the cusp, and form wells around each 'bulb' at the periphery of $\mathcal{M}$, with the lowest point being at $(-2, 0i)$. But in the Butterfly view seen in fig. 2, or in fig. 4 with $\mathcal{M}$ rendered conventionally and rotated 90 degrees; it is natural to see the Mandelbrot Set as a cosmic thermometer – where $(-2, 0i)$ represents 0° K – and this is a useful analogy I will revisit later. For the moment; we remind the reader that different forces are seen to predominate, as the universe expands and cools, and this is why we note corresponding trends which refer to both the progression of cosmological eras and the spectrum of the fundamental forces or their interactions.

Looking at Trends along the periphery of $\mathcal{M}$

The most obvious trend in the Mandelbrot Set is its asymmetric progression of form, for different Real coordinates. While a positive Real-numbered value > 0.25 goes to infinity, the negative Reals are bounded at -2 instead. Visually speaking; the cusp and the antenna spike both point toward the negative – along the Real axis. So we see a decided directionality to this figure, *along* the Real axis, though it is mirror-symmetric *about* that axis, in the direction of the (negative and positive) imaginaries. Features of $\mathcal{M}$ in regions of either half display a chirality specific to that region, and opposite to their mirror-image counterparts – which explains (under the MMC) observables like the preponderance of left-handed neutrinos, and the cosmic time direction in the early cosmos. As in the 'Spontaneous Inflation' theory of Carroll and Chen [14], cosmology based on $\mathcal{M}$ and the MMC features time that splits, or goes both ways – perhaps at the universe's inception.

The direction that is forward in time for a given sector or universe is assumed to be perceived as global time by its inhabitants – regardless of how it appears to a distant observer. We may, in fact, be forbidden to see a reverse-time branch of the universe (should it exist), because it split off before decoupling and is obscured from view, or lives in a time that hasn't happened yet. The preference nature displays for a particular chirality or handedness in its forms (and a preponderance of matter over antimatter) can thus be explained by the geometry of $\mathcal{M}$. While the Mandelbrot Set has two lobes or boundaries of opposite chirality, an observer in either time-direction would see a similarly chiral universe – where the reverse-time branch is inaccessible.

But even apart from its chiral features; the Mandelbrot Set is grossly asymmetrical from cusp to tail. It displays a progression of forms as one moves along either edge, rather than an endless repetition of similar forms. While shapes around adjoining 'bulbs' are largely similar, there is an overall trend to the changing character of the forms, as one moves along the periphery of the figure. This progression recreates many elements of natural processes that unfold in stages or sequences of events. In fact; it reproduces the character of a full range of processes, and phenomena from the smallest scale and highest temperatures or energies, to the largest cosmic scale and the lowest temperature. And the Mandelbrot Set is not only a static object, but is actually a catalog of variational and dynamical processes. As such; it depicts a full spectrum of vibratory phenomena, as well as a range of beautiful and intricate forms – worthy of note in their own right.

The symmetries in $\mathscr{M}$ represent how variations of a specific period are sustained, and the symmetry-breaking progression represents how these variations evolve. This statement reveals the essence of $\mathscr{M}$'s role as a driver of Cosmology, and I will elaborate further about this idea, later in the paper. But the decidedly lopsided shape of the Mandelbrot Set is also seen to exactly mimic 2nd law entropy, and the onset of chaos, by reproducing the Verhulst dynamics. The fundamental asymmetry of $\mathscr{M}$ is clearly illustrated by overlaying the bifurcation diagram of its generating equation, which splits wherever the edge or boundary of $\mathscr{M}$ folds back on itself, and then degenerates into chaos, but has islands of order for the mini- $\mathscr{M}$s near the Set's tail.

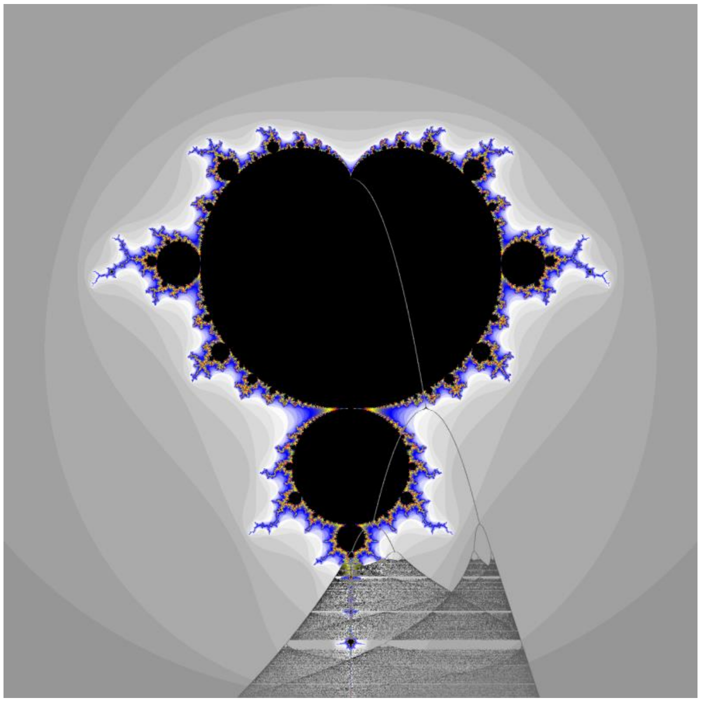

Figure 5. The Mandelbrot Set and its bifurcation diagram showing how M reproduces the Verhulst dynamic.

What is said above will be clear if the reader understands the statement about $\mathscr{M}$ being a thermometer – describing a range of energies and temperatures from those found in the primordial first fleeting moments of the Cosmos' origin to the depths of its final eon. Of course; the middle range is also clearly delineated. So the reader should rest assured that there is a range of locations represented within $\mathscr{M}$ where conditions for sustaining life forms such as our own exist. But it should also be understood that $\mathscr{M}$ reveals processes that pertain to the regime where the familiar fundamental forces were unified – or where they undergo unification – energies and spaces impossible to directly observe. So we might ask what $\mathscr{M}$ tells us about the limit of high-energy Physics near the Planck scale. The cusp at $(.25, 0i)$ is seen as the place corresponding to the first broken symmetry, which is the direction of time. That is; the flow of time is seen to proceed away from the cusp as form evolves along either edge of the figure's periphery, but this requires a choice of the left or right hand branch at that point – where time is moving in the opposite direction in the other branch.

So, following this line of reasoning, the cosmic direction of time is set at the UV limit, during the very first instant of the Big Bang. There would be an opposite branch or perhaps an array of universe timelines each possessing a unique time direction – for the higher-dimensional case including Quaternion and Octonion terms – as in Carroll and Chen's 'Spontaneous Inflation' theory [15], or perhaps similar to views of time explored in the work of Julian Barbour [16]. Some important aspects of the larger theory are summarized below.

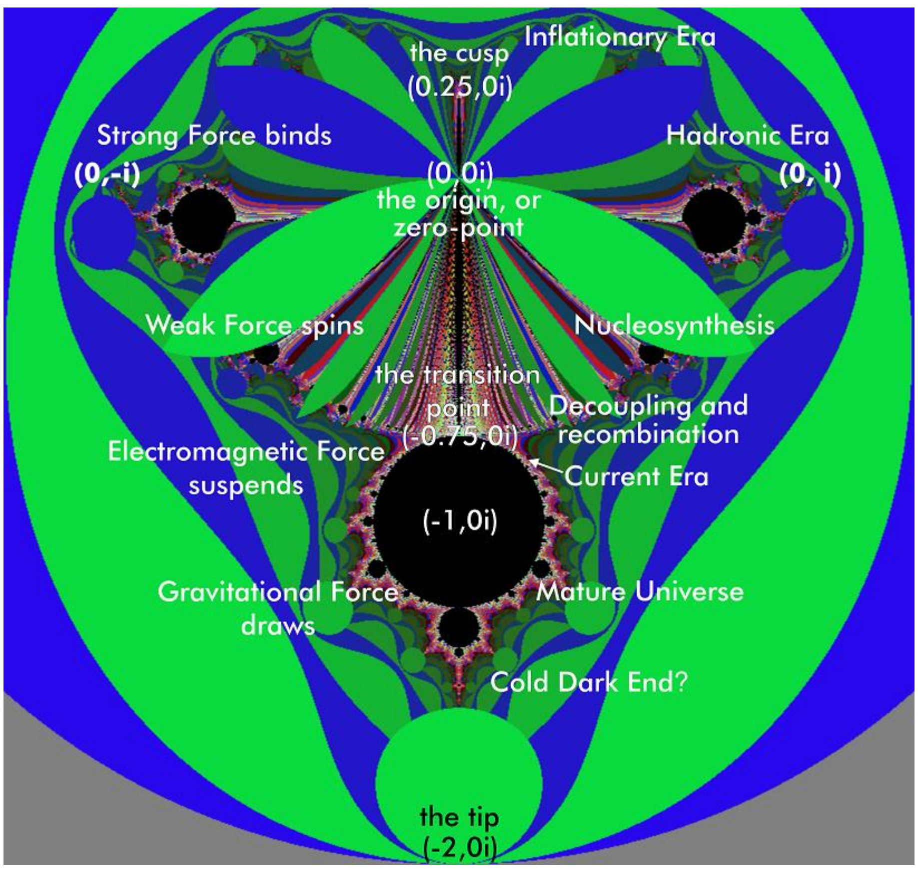

Figure 5. The Mandelbrot Set is annotated here to show the thermometer analogy and some relevant connections to Physics.

Briefly stated; there is a progression of form, as one moves outward from the cusp, that corresponds to a range of processes – going from higher to lower energy. The cusp at $(-\ 0.25, 0i)$ is the place in $\mathcal{M}$ that requires the most computational power to resolve, and arguably has the greatest logical depth [17] of any point on the Set. So the cusp requires more energy to calculate or define, than any other location on the periphery. And as noted; it is the top of the mountain a ball would roll down from, when $\mathcal{M}$ is unrolled. This, we assume, corresponds with the highest possible temperature, or the high-energy conditions one would find at the universe's inception during the Planck Era.

What about the opposite extreme? Apart from the origin, which is an immediate repeater; the easiest place to calculate is the tip of the main antenna, at $(-2, 0i)$, which is a Misiurewicz point satisfying the relation $(c^2+c)^2+c=c^2+c$. We see that the left hand side of this equation is simply an iteration of what is on the right hand side. This is the general pattern for all the Misiurewicz points, but the number of iterations represented on the left hand and right hand sides varies, depending on the cycling period and pre-period one might want to study. And again as noted; the tip of the main antenna is the low point in $\mathcal{M}$, the base of the mountain, when the Set is unfurled – so all the unspent balls are drawn there. The terminus at $(-2, 0i)$ therefore corresponds to the lowest possible temperature, in this case, 0°K or absolute zero. But in general; the trend is from representations of the highest energy phenomena nearest the cusp, and moving toward lower energy processes as one moves along the periphery greater and greater distances from the origin. Accordingly; as we trace the periphery, we move through representations of Strong, Weak, Electromagnetic, and Gravitational interactions in turn – which reflects that each of the fundamental forces assumes a defining role in successive cosmological eras.

Conclusions and Reflections

Research into this thesis has been demanding of this author, requiring extensive study, travel, and a determined effort to seek out and interact with top experts and others more well-informed than he, to find the answers needed – to make a fair determination of its value and applications. It was realized early on that there needs to be a larger context, for the theoretical structure to be relevant, and later it was discovered that others have taken steps down a road with many similar features. I investigated Computational Universe theories [18] of various flavors, because a Cosmology based on $\mathcal{M}$ has a built-in requirement for some computability constraint, or for a specific mechanism by which computation can be carried out – converting abstract Math into physical structure.

However; in my discussions with Gerard 't Hooft at FFP10, regarding his Quantum Gravity based on Cellular Automata [19]; I asked directly "what does the calculating?" and he spelled out that there don't need to be atoms of space or whatever, because "the laws of the universe do the calculating for us." This led me to examine Phil Gibbs' "Theory of Theories" [20], John Wheeler's "It from Bit" [21] and later Max Tegmark's Mathematical Universe [22], in more detail. As it turns out; the proper context for a Mandelbrot Cosmology is a very Platonic one where all the mathematical objects found by humans are seen to have already been put to use by nature – to construct the wonders of the universe. Ergo; the Mandelbrot Set is a complement to highly symmetrical forms like E8, which Garrett Lisi [23], F.D. 'Tony' Smith [24], and others see as a generator of natural law. While E8 is a symmetry-preserving object, and a catalog of symmetries, $\mathcal{M}$ is a catalog of symmetry-breaking structures – arranged in a progression.

The author has found $\mathcal{M}$ to be an apt teacher, as it gives insights into most any Physics problem, now that it is known where to look for analogous structures. In trying to summarize my work with the Mandelbrot Set, and finding connections with Cosmology, much is bound to be left out – because many unrelated facts were learned over the years. In some cases; the advances hoped for were not possible, until there were new discoveries and developments in Math, or until the author learned more of the Math and Physics necessary for further progress. In other cases; spotting a correspondence required astrophysical data unavailable when the 'Mandelbrot Butterfly' discovery was first made, or even for years after. It was only since the 'revolution in Cosmology,' which came with the discovery of accelerated expansion and other surprises near the end of the 20th century, that some of the lessons suggested by the Mandelbrot Set actually made complete sense.

It was only then that the author began making presentations to the scientific community about this theory, at Physics and Cosmology conferences, or elsewhere. What is presented here is only the broadest possible overview, and this summary represents mainly the toy model of the theory – which uses the familiar 2-dimensional form of $\mathcal{M}$ with complex numbers. The full theory involves the Mandelbrot Set that lives in Octonionic space, where the spacetime we inhabit is a Quaternionic bubble in that higher-dimensional domain. It should also be noted that this is a sketch of an idea, where future papers will explore specific exact correspondences.

The statement has been made, perhaps by David Deutsch, that if a pattern or principle is seen to apply to any part of Physics, it must ultimately apply to all of it. My personal regard for the Mandelbrot Set as a key to Cosmology is something like this. I can easily pick out ways in which the procession of forms along the periphery exactly mimics certain natural progressions and processes. Does this mean it describes everything? It is evident that $\mathcal{M}$ depicts a grand spectrum of symmetry-breaking dynamics, which appears to embody the full range from the highest temperatures to the cessation of all vibrations. So I have long wondered how deep the correspondence goes, or how many delightful correspondences might wait to be discovered. I also sense that, in some ways, the progression of forms depicted in the Mandelbrot Set has a range that goes far beyond what can exist as natural law in Physics, or what can be manifested in physical form and material substance. The real world has resolution limits, while M embodies a range of scale considerably larger than that between the largest and smallest observable structures in the universe – as illustrated when using the best software and hardware available today. On some level; for Mandelbrot Cosmology to work, it *must* be part of a larger theory involving other invariant objects of Math such as the largest exceptional group E8, and the Fischer-Greiss or Monster Group. There are symmetry-preserving objects like E8 and symmetry-breaking objects like the Mandelbrot Set, and both are cornerstones in a natural order where Math and Physics are blended.

In this manner; one can understand the fact that the Mandelbrot Set somehow mimics Cosmology as partial evidence that the Cosmos is inherently mathematical. If the universe is made of vibrations, as the String theory folks tell us, the Mandelbrot Set surely must play a part in how specific vibrations find expression in the real world, because it is an inherently spectral object which serves as a kind of resonant filter. It is the most complex object of its kind, with a far greater complexity than similar figures with a higher exponent, or more terms, where all other polynomials of higher degree or order display much greater symmetry, and lack the exquisite progression of self-similar forms present in $\mathcal{M}$. In a manner of speaking, what the Mandelbrot Set depicts along its border is the progression from one type of form to another.

One can utilize the Mandelbrot/Julia correspondence, to make animations showing the dynamics of one region morphing into those of another. In addition; Julia Sets can be given the Butterfly decorations, so the formation and dissolution of various systems, objects or quanta, and processes, can be studied – via the Mandelbrot Set, the Butterfly figure, and the related family of objects including Julia Sets. But seeing in the shoreline of $\mathcal{M}$ the advancing expression of form, the author cannot help but see a correspondence with the progression that took place to give rise to the Cosmos. He hopes this paper gives enough of an introduction so it will spark the reader's interest to explore this connection as well.

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Additional Resources

a. My poster presented at CCC-2 in Port Angeles, WA USA – Does the Mandelbrot Set offer clues to the Cosmological Evolution of Form?
b. Text pages accompanying my CCC-2 poster – An introduction and explanation
c. Preprint of my paper published in CCC-2 Proceedings, ASP Conference series, **413**, pp. 116-121; 2006 – Does the Mandelbrot Set offer clues to the Cosmological Evolution of Form?
d. My slides intended for a talk at FFP12 in Udine, Italy – What can the Mandelbrot Set Teach us about Cosmology?
e. My FQXi essay page, describing the larger context of the current paper’s thesis – How the Totality of Mathematics Shapes Physics
f. My FQXi video page, introducing the Mandelbrot Set Cosmology theory – Can the Mandelbrot Set help us understand the Cosmos?
g. The YouTube page for that video – Can the Mandelbrot Set help us understand the Cosmos?
h. The YouTube page for my companion music video – Mandelbrot Butterfly Safari