Report

Badly Behaving Photons & Spacetime as 4-surface

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Abstract

Photons in a hollow helical and conical light beam are found to behave as if they had spin 1/2. The proposed explanation is in terms of additional angular momentum like operator performing rotations around the symmetry axis of the helical beam acting as symmetries. The problem is that the definition of this operator however depends on the photon state. There are two kinds of rotations and in TGD framework the interpretation would be in terms of rotations of imbedding space on one hand and rotations at space-time surface assignable to the beam. In recent case this surface could be regarded as a 2-fold covering of Minkowski space. It seems possible to reproduce the experimental findings. Also n-fold coverings with $1/n$ spin fractionization are predicted to be possible. There is also a connection with $h_{eff} = n \times h$ hypothesis.

Keywords: Photon, anomaly, spacetime, 4 surface, spin, angular momentum.

1 Introduction

There was an interesting popular article with title *Light Behaving Badly: Strange Beams Reveal Hitch in Quantum Mechanics* (see `http://tinyurl.com/hefhdad`). The article told about a discovery made by a group of physicists at Trinity College Dublin in Ireland in the study of helical light-beams with conical geometry. These light beams are hollow and have the axis of helix as a symmetry axis. The surprising finding was that according to various experimental criteria one can say that photons have spin $S = \pm/1/2$ with respect to the rotations around the axis of the helix [1] (see `http://tinyurl.com/zoro4gz`).

The first guess would be that this is due to the fact that rotational symmetry for the spiral conical beam is broken to axial rotational symmetry around the beam axis. This makes the situation 2-dimensional. In $D = 2$ one can have braid statistics allowing fractional angular momentum for the rotations around a hole - now the hollow interior of the beam. One can however counter argue that photons with half odd integer braid spin should obey Fermi statistics. This would mean that only one photon with fixed spin is possible in the beam. Something seems to go wrong with the naive argument. It would seem that the exchange of photons does not seem to correspond to 2π rotation as a homotopy would be the topological manner to state the problem.

The authors of the article suggest that besides the ordinary conserved angular momentum one can identify also second conserved angular momentum like operator.

1. The conserved angular momentum is obtained as the replacement

$$J = L + S \rightarrow J_\gamma = L + \gamma S \ . \tag{1.1}$$

2. The eigenvalue equation for j_γ for a superposition of right and left polarizations with $S = \pm 1$

$$a_1 \times e_R exp(il_1\theta) + a_2 \times e_L exp(il_2\theta) \ , \tag{1.2}$$

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where l_i and also $s_z = \pm 1$ are integers, makes sense for

$$\gamma = \frac{(l_1 - l_2)}{2} \quad , \tag{1.3}$$

and gives eigenvalue

$$j_\gamma = \frac{l_1 + l_2}{2} \quad . \tag{1.4}$$

Since l_1 and l_2 are integers by the continuity of the wave function at 2π (even this can be questioned in hollow conical geometry) $(l_1+l_2)/2$ and $(l_1-l_2)/2$ are either integers or half integers. For $l_1-l_2=1$ the one has $J_\gamma = J_{1/2} = L + S/2$, which is half odd integer. The stronger statement would be that 2-D $S_\gamma = S/2$ is half-odd integer.

There is an objection against this interpretation. The dependence of the angular momentum operator on the state of photon implied by $\gamma = (l_1 - l_2)/2$ is a highly questionable feature. Operators should not depend on states but define them as their eigenstates. Could one understand the experimental findings in some different manner? Could the additional angular momentum operator allow some natural interpretation? If it really generates rotations, where does it act?

2 TGD based model

In TGD framework this question relates interestingly to the assumption that space-time is 4-surface in $M^4 \times CP_2$. Could X^4 and M^4 correspond to the two loci for the action of rotations? One can indeed have two kinds of photons. Photons can correspond to space-time sheets in $M^4 \times CP_2$ or they can correspond to space-time sheets topologically condensed to space-time surface $X^4 \subset M^4 \times CP_2$. For the first option one would have ordinary quantization of angular momentum in M^4. For the second option quantization in X^4 angular momentum, which using the units of M^4 angular momentum could correspond to half-integer or even more general quantization.

1. For the first (photons in M^4) option angular momentum $J(M^4) = L(M^4) + S(M^4)$ acts at point-like limit on a wave function of photon in M^4. $J(M^4)$ acts as a generator of rotations in M^4 should have the standard properties: in particular photon spin is $S = +/-1$.

2. For topologically condensed photons at helix the angular momentum operator $J(X^4) = L(X^4) + S(X^4)$ generates at point-like limit rotations in X^4. If M^4 coordinates - in particular angle coordinate ϕ around helical axis - are used for X^4, the identifications

$$J(X^4) = kJ(M^4) \quad , \quad L(X^4) = kL(M^4) \quad , \quad S(X^4) = kS(M^4) \quad . \tag{2.1}$$

are possible.

3. In the recent case X^4 corresponds to effectively a helical conical path of photon beam, which is effectively 2-D system with axial $SO(2)$ symmetry. The space-time surface associated with the helical beam is analogous to a covering space of plane defined by Riemann surface for $z^{1/n}$ with origin excluded (hollowness of the spiral beam is essential since at z-axis various angles ϕ correspond to the same point and one would obtain discontinuity). It takes n full turns before one gets to the original point. This implies that $L(X^4) = kL(M^4)$ can be fractional with unit $\hbar/n$ meaning $k = 1/n$ when the angle coordinate of M^4 serves as angle coordinate of X^4.

4. For $n = 2$ one has $k = 1/2$ and 4π rotations in Minkowski space interpreted as shadows of rotations at X^4 must give a phase equal to unity. This would allow half integer quantization for $J(X^4), L(X^4)$ and $S(X^4)$ of photon in M^4 units. $S(X^4)$ corresponds to a local rotation in tangent space of X^4. The braid rotation defined by a path around the helical axis corresponds to a spin rotation and by $k = 1/2$ to $S(X^4) = S(M^4)/2 = 1/2$. Hence one has effectively $S(M^4) = \pm 1/2$ for the two circular polarizations and thus $\gamma = \pm 1/2$ independently of l_i: in the above model $\gamma = (l_1 - l_2)/2$ can have also other values. Now also other values of n besides $n = 2$ are predicted.

   l_i can be both integer and half odd integer valued. One can reproduce the experimental findings for integer valued l_1 and l_2. One has $j = l_1 + 1/2 = l_2 - 1/2$ from condition that superpositions of both right and left-handed spiral photons are possible. If j is half-odd integer, $l_1 + l_2 = 2j$ is odd integer. For instance, $S(X^4) = 1/2$ gives $l_1 - l_2 = -1$ consistent with integer/half-odd integer property of both l_1 and l_2. For $j = 1/2$ one has $l_1 + l_2 = 1$ and $l_1 - l_2 = -1$ giving $(l_1, l_2) == (0, 1)$.

5. Is there something special in $n = 2$. In TGD elementary particles have wormhole contacts connecting two space-time sheets as building bricks. If the sheets form a covering of M^4 singular along plane M^2 one has $n = 2$ naturally.

One can worry about many-sheeted statistics. The intuitive view is that one just adds bosons/fermions at different sheets and each sheet corresponds to a discrete degree of freedom.

1. Statistics is not changed to Fermi statistics if the exchange interpreted at X^4 corresponds to $n \times 2\pi$ rotation. For $n = 2$ a possible modification of the anti-commutation relations would be doubling of oscillator operators assigning $a_k(i)$, $i = 1, 2$ to the 2 sheets and formulating braid anti-commutativity as

$$\begin{array}{lll} \{a_k(1), a_l(2)\} = 0 \ , & \{a_k^\dagger(1), a_l^\dagger(2)\} = 0 \ , & \{a_k^\dagger(1), a_l(2)\} = 0 \ . \\ [a_k(i), a_l(i)] = 0 & \left[a_k^\dagger(i), a_l^\dagger(i)\right] = 0 & \left[a_k^\dagger(i), a_l(i)\right] = \delta_{k,l} \ . \end{array} \tag{2.2}$$

   This would be consistent with Bose-Einstein statistics. For n-sheeted case the formula replacing pair $(1, 2)$ with any pair $(i, j \neq i)$ applies. One would have two sets of mutually commuting (creation) operators and these sets would anti-commute and Bose-Einstein condensates seem to be possible.

2. One can worry about the connection with the hierarchy of Planck constants $h_{eff} = n \times h$, which is assigned with singular n-sheeted covering space [2]. The 3-D surfaces defining ends of the covering at the boundaries of causal diamond (CD) would in this case co-incide. This might be the case now since the photon beam is assumed to be conical helix. Space-time surface would be analogous to n 3-D paths, which co-incide at their ends at past and future boundaries of CD.

   Does the scaling of Planck constant by n compensate for the fractionization so that the only effect would be doubled Bose-Einstein condensate. It woud seem that these condensates need not have same numbers of photons. The scaling of cyclotron energies by n is central in the application of $h_{eff} = nh$ idea. It could be interpreted by saying that single boson state is replaced with n-boson state with the same cyclotron frequency but n-fold energy.

3. In the fermionic case on obtain n additional degrees of freedom and ordinary single fermion state would be replaced with a set of states containing up to n fermions. This would lead to a kind of breakdown of fermion statistics possibly having interpretation in terms of braid statistics. And old question is whether one could understand quark color as $h_{eff}/h = n = 3$ braid statistics for leptons. At the level of CP_2 spinors em charge corresponds to sum of vectorial isospin and of anomalous color hypercharge which is for leptons $n = 3$ multiple of that for quarks. This could be perhaps interpreted in terms of scaling in hypercharge degree of freedom due to 3-sheeted covering. This picture does not seem however to work.

To sum up, also M^4 angular momentum and spin make sense and are integer valued but for the system identifiable as topological condensed photon plus helix rather than topological condensed photon at helix. Many-sheeted space-time can in principle rise to several angular momenta of this kind. Symmetry breaking go $SO(2)$ subgroup is however involved. The general prediction is $1/n$ fractionization.

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