Article

Five-Dimensional Exact Bianchi Type-I Cosmological Models in a Scalar-Tensor Theory

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Abstract

The present study deals with a spatially homogeneous and anisotropic five dimensional Bianchi type-I cosmological models filled with perfect fluid in the frame work of scalar-tensor theory proposed by Seaz and Ballester (1985). Two different physically viable models ($n \neq 0$, $n = 0$) of the universe are obtained by using a special law of variation for hubble's parameter which gives a constant value of deceleration parameter. The Einstein's field equations are solved exactly and solutions are obtained. Also physical and kinematical properties of the models are discussed.

Keywords: Five-dimensional, Bianchi Type-I, spacetime, Hubble's parameter, deceleration parameter, cosmological model, scalar field.

1. Introduction

In recent years, there has been considerable interest in string cosmology. Many authors have obtained cosmic string in different Bianchi type space times [Pradhan et al. (2009), Amirhashci and Zainuddin (2010) and Tripathi et al. (2009, 2010)]. The simplest anisotropic models of the universe are Bianchi type-I homogeneous models whose spatial section are flat but the expansion or contraction rate are directional dependent. The advantages of these anisotropic models are that they have a significant role in the description of the evolution of the early phase of the universe and they help in finding more general cosmological models than the isotropic FRW models. Saha and Visinescu (2010) and Saha et al.(2010) have studied Bianchi type-I models with cosmic string in presence of magnetic flux. Recently, the Einstein's field equations have been solved for massive string by applying a variation law for generalized Hubble's parameter in Bianchi-I space-time by Pradhan and Chouhan (2011).

In last few decades, many researchers are interested in the alternative theories of gravitation since from the birth of the Einstein's general theory of relativity does not seem to resolve some of the important problems in cosmology like dark matter or missing matter problem. Saez and Ballester (1985) have developed a theory in which the metric is coupled with a dimensionless scalar field ϕ and the $\phi-$coupling gives a satisfactory description of the weak fields. Scalar-

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tensor theory helps to solve the problem in non-flat FRW cosmologies. Many authors have investigated cosmological models within the framework of Saez-Ballester's scalar-tensor theory of gravitation. Single and Agrawal (1991,92) have studied Bianchi type I to IX models in Scalar tensor. Shri Ram and Singh (1995) have obtained spatially homogeneous and locally rotationally symmetric (LRS) solutions which admit a Bianchi-I group of motions on hypersurfaces *t*=constant. Shri Ram and Tiwari (1998) have investigated inhomogeneous plane symmetric models. Singh and Shri Ram (2003) have presented a spatially homogeneous and isotropic FRW model with zero curvature. Reddy (2003) has obtained an exact Bianchi type-I string cosmological model in the Seaz Ballester's theory. Mohanty and Sahu (2003,04) have studied Bianchi type-VI_0 and Bianchi type-I models in this theory. Reddy et. at. (2006) have obtained exact solutions for a spatially homogeneous and LRS Bianchi-I space time with constant deceleration parameters in the Saez Ballester's scalar tensor theory. Recently G. C. Samanta et al. (2012) have obtained five dimensional Bulk Viscous string cosmological models in Saez and Ballester theory of gravitation.

The study of higher dimensional cosmological models is motivated mainly by the possibility of geometrically unifying the fundamental interactions of the universe. String theory unifies all the elementary interactions including gravity. The study of higher dimensional physics is important because of several prominent results obtained in the development of the superstring and other field theory. Krori et al. (1994) constructed a Bianchi type-I string cosmological model in higher dimensional space time and obtained that matter and string coexist throughout the evolution of the universe. Several authors constructed five dimensional cosmological models in various aspects.

Thus in the present paper, we have obtained exact solutions of five dimensional anisotropic Bianchi type-I space-time in the scalar tensor theory proposed by Seaz and Ballester (1985) on the lines of Suresh Kumar and C. P. Singh (2008).

The paper is organized as follows : In section-2, we give a introduction about the field equations. In Sec.3, we deal with an exact solution of the field equations with perfect fluid. In section-4,5, we have discussed the cosmological model with physical behavior for both the models and in the last section-6, we summarize and conclude the results.

2. The metric and field equations in V_5

We consider five dimensional spatially homogeneous and anisotropic Bianchi type-I metric

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)(dy^2 + dz^2) + C^2(t)du^2 \tag{1}$$

where A, B and C are functions cosmic time *t* alone.

The field equation given by Saez and Ballester (1985) for the combined scalar and tensor field is

$$G_{ij} - \omega\phi^m(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) = -T_{ij} \tag{2}$$

and scalar field ϕ satisfies the equation

$$2\phi^m\phi^i_{;j} + m\phi^{m-1}\phi_{,k}\phi^{,k} = 0 \tag{3}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor,

ω and m are constant , T_{ij} is stress energy tensor of matter,
comma and semicolon denote partial and covariant differentiation respectively.

The energy momentum tensor T_{ij} for perfect fluid distribution is taken as

$$T_{ij} = (\rho + p)v_i v_j + pg_{ij} \tag{4}$$

where p is pressure and ρ is the energy density for a fluid $v^i = (0,0,0,0,1)$ is the five velocity of the particle and $v^i v_i = 1$.

In a co-moving coordinate system, the field equations (2) and (3) for the anisotropic Bianchi type-I space time (1) in case of (4) as

$$\frac{2\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{2\dot{B}\dot{C}}{BC} + \frac{\dot{B}^2}{B^2} = p + \frac{\omega}{2}\phi^m\dot{\phi}^2, \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = p + \frac{\omega}{2}\phi^m\dot{\phi}^2, \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = p + \frac{\omega}{2}\phi^m\dot{\phi}^2, \tag{7}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{2\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}^2}{B^2} = \rho + \frac{\omega}{2}\phi^m\dot{\phi}^2, \tag{8}$$

$$\ddot{\phi} + \dot{\phi}(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C}) + \frac{m}{2}\frac{\dot{\phi}^2}{\phi} = 0. \tag{9}$$

The energy conservation equation

$$T^{ij}_{;j} = 0\,, \tag{10}$$

leads to the expression

$$\dot{\rho} + (\rho + p)[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C}] = 0\,. \tag{11}$$

which is a consequence of the field equations (5)-(9) are five equations involving six unknown A, B, C, ρ , p and ϕ .

3. Solutions of the Einstein's field equations in V_5

Einstein's field equations (5)-(9) are a coupled system of higher non-linear differential equation. The solutions of the field equations are solved by applying a variation law for generalized Hubble parameter proposed by Berman (1983) in space-time (1) which gives a constant deceleration parameters. Many authors have considered cosmological models with constant deceleration parameter. Reddy et al. (2006,07) have presented LRS Bianchi type-I models with constant deceleration parameter in scalar tensor and scale covariant theories of gravitation. Also Suresh Kumar and C. P. Singh (2006,07) have investigated LRS Bianchi type-II models with constant deceleration parameter in general relativity, Guth's inflationary theory and self creation theory of gravitation.

Recently Suresh Kumar and C. P. Singh (2007) have obtained a exact solutions for a spatially homogeneous and anisotropic Bianchi-type-I space-time with perfect fluid in general relativity by applying a special law of variation for Hubble's parameters that yield a constant value of deceleration parameter.

Using the special law of variation for the Hubble parameter given by Berman(1983), gives a constant value of deceleration parameter. Hence, the law read as

$$H = Da^{-n} = D[AB^2C]^{\frac{-n}{4}} \tag{12}$$

where $D > 0$ and $n \geq 0$ are constant.

Considering $a = (AB^2C)^{\frac{1}{4}}$, $V = a^4 = AB^2C$ as the average scale factor and the volume scale factors of the anisotropic Bianchi-I space-time in V_5 and Hubble parameter is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{4}[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C}]\,. \tag{13}$$

From (12) and (13) we get,

$$\frac{1}{4}[\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}+\frac{\dot{C}}{C}]=l[AB^2C]^{\frac{-n}{4}}\ . \tag{14}$$

Integration of (14) gives

$$a=(nDt+c_1)^{1/n}\ , \quad (n\neq 0) \tag{15}$$

$$a=c_2e^{Dt}\ , \quad (n=0) \tag{16}$$

where c_1 and c_2 are constant of integration.

It is mention here that we have used $H=Da^{-n}$, $D>0$, $n\geq 0$ to get the above equation. Thus we obtain two values of the average scale factor corresponding to two different models of the universe.

The value of deceleration parameter (q), is then found to be

$$q=-\frac{a\ddot{a}}{\dot{a}^2}\ , \tag{17}$$

Substituting (15) into (17), we get

$$q=(n-1) \tag{18}$$

which is constant value of deceleration parameter.

Subtracting (8) from (9) and taking integral of the resulting equation two times, we get

$$\frac{A}{B}=d_1\exp[x_1\int a^{-4}dt]\ , \tag{19}$$

$$\frac{A}{C}=d_2\exp[x_2\int a^{-4}dt] \tag{20}$$

$$\frac{B}{C}=d_3\exp[x_3\int a^{-4}dt]\ , \tag{21}$$

where d_1, d_2, d_3 and x_1, x_2, x_3 are constants of integration.

From equations (19)-(21), the metric functions can be written as

$$A = a_1 a \exp[b_1 \int a^{-4} dt], \tag{22}$$

$$B = a_2 a \exp[b_2 \int a^{-4} dt], \tag{23}$$

$$C = a_3 a \exp[b_3 \int a^{-4} dt], \tag{24}$$

where $a_1 = (d_1^2 d_2)^{1/4}, \quad a_2 = (d_1^{-1} d_3)^{1/4}, \quad a_3 = (d_2^{-1} d_3^{-2})^{1/4},$ (25)

and $b_1 = \dfrac{2x_1 + x_2}{4}, \quad b_2 = \dfrac{x_3 - x_1}{4}, \quad b_3 = -(\dfrac{x_2 + 2x_3}{4})$.

Satisfy the following relation

$$a_1 a_2^2 a_3 = 1, \qquad b_1 + 2b_2 + b_3 = 0. \tag{26}$$

The integral of (9) gives

$$\phi(t) = \left[\frac{h(m+2)}{2} \int a^{-4} dt \right]^{\frac{2}{m+2}} \tag{27}$$

where h is constant of integration.

4. Five dimensional Model of the Universe when $n \neq 0$

This model is based on the exact solutions of five dimensional Bianchi type-I space-time filled with perfect fluid.

Using (15) in (22-24), we obtain the metric functions as

$$A(t) = a_1 (nDt + c_1)^{1/n} \exp[\frac{b_1 (nDt + c_1)^{\frac{n-4}{n}}}{D(n-4)}], \tag{28}$$

$$B(t) = a_2 (nDt + c_1)^{1/n} \exp[\frac{b_2 (nDt + c_1)^{\frac{n-4}{n}}}{D(n-4)}], \tag{29}$$

$$C(t) = a_3 (nDt + c_1)^{1/n} \exp[\frac{b_3 (nDt + c_1)^{\frac{n-4}{n}}}{D(n-4)}]. \tag{30}$$

Substituting (15) in (27), the scalar field is given by

$$\phi(t) = \left[\frac{h(m+2)}{2D(n-4)} \right]^{\frac{2}{m+2}} (nDt + c_1)^{\frac{2(n-4)}{n(m+2)}}. \tag{31}$$

Putting (28-30) in (7) and (8), the pressure and energy density of the model is given by

$$p = D^2(3n-6) - [b_1^2 + 3b_2^2 + 2b_1b_2 - \frac{1}{2}\omega h^2](nDt + c_1)^{-\frac{8}{n}}, \tag{32}$$

$$\rho = -6D^2(nDt + c_1)^{-2} - (nDt + c_1)^{-\frac{8}{n}}[2b_1b_2 + 2b_2b_3 + b_1b_3 + b_2^2 + \frac{1}{2}\omega h^2] \tag{33}$$

The solutions (28-30) satisfy the energy conservation equation (11) identically and hence represent exact solutions of the Einstein's field equation (5-9).

The directional Hubble factors $H_i (i = 1,2,3,4)$ in the direction of x, y, z and u read as

$$H_i = D(nDt + c_1)^{-1} + b_1(nDt + c_1)^{-4/n} \tag{34}$$

The expansion scalar θ is given by

$$\theta = 4H = 4D(nDt + c_1)^{-1} \tag{35}$$

The anisotropy parameter A is define as

$$A = \frac{1}{4D^2}(b_1^2 + 2b_2^2 + b_3^2)(nDt + c_1)^{\frac{2n-8}{n}} \tag{36}$$

The special volume (V) are found to be

$$V = (nDt + c_1)^{1/n}. \tag{37}$$

$$\sigma^2 = \frac{1}{4}[(\frac{3}{2}b_1^2 + 2b_2^2 + \frac{3}{2}b_3^2 - 2b_1b_2 - b_1b_3 + 2b_2b_3)(nDt + c_1)^{-\frac{8}{n}} + 4D^2(nDt + c_1)^{-2}$$

$$+ 4D^2(b_2 + b_3)(nDt + c_1)^{\frac{-n-4}{n}}]. \tag{38}$$

From the above results, it can be seen that the spatial volume is zero at $t = -(c_1 / nD)$, and expansion scalar is infinite, which shows that the universe starts with zero volume with infinite rate of expansion. The scalar factors also vanishes at $t = -(c_1 / nD)$ and singularity obtained at initial stage. As t increases, the scale factors and spatial volume increases but expansion scalar decreases. Thus the rate of expansion slows down with increase in time. Also $\phi, \rho, p, H_1, H_2, H_3, H_4, A$ and σ^2 decrease at t increases. As $t \to \infty$, scale factors and volume become infinite whereas $\phi, \rho, p, H_1, H_2, H_3, H_4, A$ and σ^2 tend to zero. Therefore, the model would essentially give an empty universe for large time *t*. The ratio σ / θ tend to zero as $t \to \infty$ provided $n < 4$. Therefore, the model approaches isotropy for large values of *t*. Hence the model represents shearing, nonrotating and expanding model of the universe at late times.

5. Five dimensional Model of the Universe when $n = 0$

This model is based on the exact solutions of five dimensional Bianchi type-I space-time filled with perfect fluid. Using (17) in (23-25), we obtain the metric functions as

$$A(t) = a_1 c_2 \exp[Dt - \frac{b_1}{4DC_2^4} e^{-4Dt}], \tag{39}$$

$$B(t) = a_2 c_2 \exp[Dt - \frac{b_2}{4DC_2^4} e^{-4Dt}], \tag{40}$$

$$C(t) = a_3 c_2 \exp[Dt - \frac{b_3}{4DC_2^4} e^{-4Dt}]. \tag{41}$$

The scalar field is given by

$$\phi(t) = \left[\frac{h(m+2)}{8Dc_2^4}\right]^{\frac{2}{m+2}} \exp(-\frac{8Dt}{m+2}). \tag{42}$$

Putting (28-30) in (7) and (8), the pressure and energy density of the model is given by

$$p = 6D^2 + [b_1^2 + 3b_2^2 + 2b_1 b_2 - \frac{1}{2}\omega h^2]c_2^{-8} e^{-8Dt}, \tag{43}$$

$$\rho = 6D^2 + [(2b_1 b_2 + 2b_2 b_3 + b_1 b_3 + b_2^2) - \frac{1}{2}\omega h^2]c_2^{-8} e^{-8Dt} \tag{44}$$

The solutions (28-30) satisfy the energy conservation equation (11) identically and hence represent exact solutions of the Einstein's field equation (5-9).

The directional Hubble factors $H_i (i = 1,2,3,4)$ in the direction of x, y, z and u read as

$$H_i = D + b_i c_2^{-4} e^{-4Dt}, \quad (i = 1,2,3,4) \tag{45}$$

The expansion scalar θ is given by

$$\theta = 4H = 4D. \tag{46}$$

The anisotropy parameter A is define as

$$A = \frac{1}{4D^2}(b_1^2 + 2b_2^2 + b_3^2)c_2^{-8} e^{-8Dt}. \tag{47}$$

The special volume (V) are found to be

$$V = c_2 e^{Dt}. \tag{48}$$

$$\sigma^2 = [D^2 + (3b_1^2 + 4b_2^2 + 3b_3^2 - 4b_1 b_2 - 2b_1 b_3 + 4b_2 b_3)\frac{c_2^{-8}}{8} e^{-8Dt} + D(b_2 + b_3)c_2^{-8} e^{-4Dt}. \tag{49}$$

From the above results, it can be seen that the spatial volume, scale factors, scale field, pressure, energy density and cosmological parameters are constant at $t = 0$. Thus universe starts evolving with a constant volume and expands with exponential. As t increases, the scale factors and spatial volume increases exponentially while the scale field, pressure, energy density, anisotropy parameter and shear scalar decrease. It is to noted that the expansion scalar is constant throughout the evolution of universe and therefore the universe exhibits uniform exponential expansion in this model.

As $t \to \infty$, the scale factors and volume of the universe become infinitely large whereas the scalar field, anisotropy parameter and shear scalar tend to zero. The pressure, energy density and Hubble's factors become constants such that $p = -\rho$. The model approaches isotropy for large time *t*. Thus the model representing shearing, nonrotating and expanding model of the universe with a finite start approaching to isotropy at late times. For $n = 0$, $q = -1$, this value of deceleration parameter leads to $dH/dt = 0$, gives the greatest value of Hubble's parameters and the fastest rate of expansion of the universe. Therefore the solutions presented in this model are consistent with the observations.

6. Conclusion

In the present paper, we have obtained a spatially homogeneous and anisotropy Bianchi type-I space-time using scalar tensor theory of gravitation proposed by Saez and Ballester (1985). These field equations have been solved using a special law of variation of Hubble's parameter that yields a constant value of deceleration parameter. Two models have been obtained $(n \neq 0, n = 0)$ also some important cosmological parameters have been obtained for both the models and physical behavior of the models is discussed in detail.

For $n \neq 0$, scale factors and volume vanishes. As $t \to \infty$, the pressure, energy density and scalar field become negligible whereas the scale factors and spatial volume become infinitely large.

For $n = 0$, density being finite. The universe exhibit exponential expansion and expand uniformly. Both models represent shearing, non-rotating and expanding universe, which approaches to isotropy for large values of t. The scalar field decreases to zero as $t \to \infty$ in both the models. If we assume $h = 0$ then solution reduce to the solutions in general relativity [8] and the Saez-Ballester's scalar-tensor theory of gravitation tends to standard general theory of relativity in al respect. The solutions obtained in the models are found to be consistent with the recent observations. Finally the solutions presented in this paper are new and may be useful for better understanding of the evolution of universe in Bianchi type-I space-time within the framework of Saez-Balleste's scalar tensor theory of gravitation.

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