Stability of Strange Quark Matter Cosmology in Modified Theory of Gravitation

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Abstract
In this paper, we investigate the Bianchi type I metric of the Kasner form with strange quark matter coupled to the domain walls in the context of $f(R,T)$ modified theory of gravitation. Three different functional forms of the function $f(R,T)$ are considered. Stability of the investigated models is analyzed. It is found that the domain walls are stiff and the models represent the early stages of the evolution of the universe. It is also observed that the `gravitation affects on the behavior of the domain walls and the stability of the models.

Keywords: Strange quark matter, domain walls, $f(R,T)$ theory.

1. Introduction
The mysteries of the universe have always invited the curiosity of the human being from the centuries. Nowadays, the large scale structure of the universe has been an active field of the researchers. In spite of a number of intellectual attempts, exact physical situation of the universe at the early stages of the formation of the universe is still unknown. According to the grand unified theories, the stable topological defects are formed at the early phases of the formation of the universe. It is assumed that when the discrete symmetry of the universe is broken spontaneously, the stable topological defects are formed [1]. Cosmic strings, domain walls, monopoles are cosmologically important topological defects. The domain walls play a vital role in the galaxy formation of the universe.

Galaxies are formed during phase transition after recombination of matter and radiation with the help of domain walls [2]. Vilenkin [3], Reddy et al.[4], Katore et al.[5], Goetz [6] are some of the authors who have investigated several aspects of the domain walls in different contexts. Another important phase transition of the universe is the Quark Gluon Plasma to hadron gas transition when the cosmic temperature was~200 Mev. It was conjectured that at high temperature the color charge is screened and the corresponding phase of matter was named Quark Gluon

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Plasma(QGP). In QGP, the electric charges are replaced by the color charges of quarks and gluons, mediating the strong interaction among them [7]. The quark matter is formed in the quark hadron phase transition in the early universe. The quark models are based on the phenomenological bag model. Assuming quarks are massless and non-interacting, the equation of state for strange quark matter is [8]

$$P_m = \frac{1}{3}(\rho_m - 4B_c).$$

(1)

The total energy density is

$$\rho_m = \rho_q + B_c.$$  

(2)

The total pressure is

$$P_m = P_q - B_c,$$  

(3)

where $P_q = \frac{1}{3}\rho_q$ is the quark pressure. $\rho_q$ is the quark energy density and $B_c$ is the bag constant.

Recently, the quark gluon plasma is created as perfect fluid with following equation of state [8]

$$P_m = (\gamma - 1)\rho_m, 1 \leq \gamma \leq 2.$$  

(4)

Of late, a number of cosmological observations indicate that the universe is accelerating and expanding [9, 10]. There are two approaches viz. modification of the gravitation and the dark energy, to explain this accelerated expansion of the universe. One approach is to modify the gravitational theory. This is attracting considerably to the researchers in this field. The late-time cosmic acceleration of the universe can be explained by $f(R)$ gravity [11]. Among the various modified theories of gravitation, there is an interesting extension of the standard general relativity, the $f(R,T)$ modified theory of gravitation where the Lagrangian is an arbitrary function of $R$ and $T$ [12]. Recently, Houndjo [13] have investigated the cosmological reconstruction in the $f(R,T)$ modified theory of gravitation. Reconstruction of cosmological models in the $f(R,T)$ theory of gravitation is also studied by Jamil et al. [14]. This motivated us to investigate strange quark matter coupled to the domain walls with the Bianchi type I metric of the Kasner form in the context of the $f(R,T)$ theory of gravitation. The paper is organized as follows: In the section 2, we present metric and field equations. The sections 3, 4 and 5 are devoted to the analysis of the three different functional forms of $f(R,T)$ respectively. The section 6 concludes the finding of the paper.

2. Metric and Field Equations

The most important cosmological observational discovery of the microwave background radiation is the existence of an anisotropic phase in the evolution of the universe. This stimulates to study models of the universe with anisotropic background space time structure. The
homogeneous but anisotropic perturbative form of Friedmann model for $k = 0$ gives Bianchi types I or VII$ _0$. Bianchi type I is the simplest cosmological model and corresponds to the universe expanding at different rates in the three orthogonal directions in the Euclidean space section. We consider the Bianchi type I metric of the Kasner form as

$$ds^2 = dt^2 - t^{2p_1}dx^2 - t^{2p_2}dy^2 - t^{2p_3}dz^2,$$

where $p_1, p_2, p_3$ are the parameters.

It is widely accepted that Bianchi type models have significant role in the description of the evolution of the early phase of the universe. These models help in finding more general cosmological models than the isotropic FRW models. The homogeneous and anisotropic Bianchi type I cosmological models have been considered by many authors [19, 20].

The field equations of the $f(R, T)$ modified theory of gravitations are

$$f_R(R, T)R_{ij} - \frac{1}{2} f(R, T)g_{ij} + (g_{ij} \nabla_i \nabla j - \nabla_i \nabla_j) f_k(R, T) = 8\pi T_{ij} - f_T(R, T) g_{ij} - f_T(R, T) \Theta_{ij},$$

where $f(R, T)$ is an arbitrary function of the Ricci Scalar $R$ and the trace $T$ of the stress energy momentum tensor $T_{ij}$. $f_R(R, T) = \frac{df(R, T)}{dR}$, $f_T(R, T) = \frac{df(R, T)}{dT}$ and all other symbols have their usual meaning as in the Riemannian geometry. The functional $f(R, T)$ depends on the nature of the matter field. The choice of the functional $f(R, T)$ is arbitrary and hence, different choices of the functional lead to the different models. Harko et al.[12] studied that due to coupling of matter and geometry, this gravity model depends on a source term, representing the variation of the matter-stress energy tensor with respect to the metric. Thus, in this gravity theory, cosmic acceleration is achieved not only from geometrical contribution but also from the matter content. Along with this, arbitrary choice of the functional $f(R, T)$ will make arbitrary to find the appropriate model resembling to cosmological observations. They have considered the following three different classes of the $f(R, T)$ gravity models

$$f(R, T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases}$$

In this work, we have considered the following three different functional forms

$$f(R, T) = \begin{cases} R + 2\mu T \\ R + bR^n + \mu T \\ \lambda R + \lambda T \end{cases}$$
The energy momentum tensor of the domain walls is taken as

$$T_{ij} = \rho (g_{ij} + w_i w_j) + P g_{ij},$$

(7)

where \( \rho \) is the energy density, \( P \) stands for pressure, \( w^i \) is the four velocity vector satisfying \( w_i w^i = -1 \). Then, we have

$$T_1^1 = T_2^2 = T_3^3 = \rho, T_4^4 = -P, T = 3 \rho - P.$$ 

(8)

Zeldovich et al have pointed out the important gravitational effect to be expected of domain walls when the universe exceeded the critical temperature. The surface energy is the inertial mass of the wall and the motion of a domain wall are controlled by its structure tension [21].

This form of the domain wall includes the quark matter. It is described by \( \rho_m = \rho_q + B_c \) and \( P_m = P_q - B_c \) as well as domain wall tension \( \sigma_d \) i.e. \( P = P_m - \sigma_d \) and \( \rho = \rho_m + \sigma_d \) [8]. It has been pointed out that a star which is smaller than neutron star could be a quark star. It is supported by degenerate pressure of quark matter. It is assumed that, such quark stars contain quark matter in the core surrounded by region and are surrounded by harmonic matter [22]. Rao and Neelima [23] have studied axially symmetric space time with strange quark matter attached to string cloud in self creation theory and general relativity. Caglar and Aygun [24] have investigated string cloud and domain walls with quark matter for a higher dimensional FRW universe in self creation cosmology.

### 3. Model I

In this section, we consider the following functional form of the function

$$f(R, T) = R + 2 \mu T.$$ 

(9)

This functional form is investigated by Harko et al. [12]. Using this form, we get solvable, simplest field equations. This functional led to cosmological model which is equivalent to an effective cosmological model. The term \( 2 \mu T \) in the gravitational action modifies the gravitational interaction between matter and curvature, replacing \( G \) by a running gravitational coupling parameter [12]. One can find considerable cosmological models in the literature with this function.

The field equations (6) using equations (5), (8) and (9) explicitly written as

$$\left[ p_1 (s-1) - \frac{1}{2} (s^2 - 2s + \theta) \right] t^2 = (8 \pi + 5 \mu) \rho + \mu P,$$

(10)

$$\left[ p_2 (s-1) - \frac{1}{2} (s^2 - 2s + \theta) \right] t^2 = (8 \pi + 5 \mu) \rho + \mu P,$$

(11)
\[
\left[ p_3(s-1) - \frac{1}{2}(s^2 - 2s + \theta) \right] t^{-2} = (8\pi + 5\mu)\rho + \mu P,
\]
\[
\left[ \frac{1}{2}(\theta - s^2) \right] t^{-2} = -(8\pi + \mu)P - 3\mu \rho.
\]

From equations (10)-(12), we have
\[
p_1 = p_2 = p_3 = p'.
\]

Using equations (14) and equations (10)-(13), we obtain the energy density (\(\rho\)) and pressure (\(P\)) as follows
\[
\rho = \frac{[8\pi + \mu](s-1)p' - 4\pi s^2 - \mu s^2 + 8\pi s + \mu s - 4\pi \theta]}{64\pi^2 + 48\pi \mu + 8\mu^2} t^{-2},
\]
\[
P = \frac{[3\mu(s-1)p' + 3\mu s + \mu s^2 + 4\pi s^2 - 4\mu \theta - 4\pi \theta]}{64\pi^2 + 48\pi \mu + 8\mu^2} t^{-2},
\]

From equation (15), it is clear that the energy density is decreasing function of time. It is large near Big-Bang singularity and as time increase, it becomes very small.

From equations (1) and (15), we have the following expressions of the quark energy density (\(\rho_q\)), quark pressure (\(P_q\)) and tension of the domain walls (\(\sigma_d\))
\[
\rho_q = \frac{[3(8\pi + \mu)(s-1)p' - 12\mu \theta + 24\pi s + 12\mu s - 24\pi \theta]}{256\pi^2 + 192\pi \mu + 32\mu^2} t^{-2} - B_c,
\]
\[
P_q = \frac{[(8\pi + \mu)(s-1)p' - 4\mu \theta + 8\pi s + 4\mu s - 8\pi \theta]}{256\pi^2 + 192\pi \mu + 32\mu^2} t^{-2} - \frac{2}{3} B_c,
\]
\[
\sigma_d = \frac{[(8\pi + \mu)(s-1)p' - 16\pi s^2 - 4\mu s^2 + 12\mu \theta - 8\pi s - 8\mu s + 8\pi \theta]}{256\pi^2 + 192\pi \mu + 32\mu^2} t^{-2} + B_c.
\]

From figure (1), it is clear that the quark energy density is decreasing function of cosmic time. It is large near the Big Bang singularity; afterwards, it decreases gradually with increasing time and eventually, tends to be zero. The tension of the domain walls is negative. Thus, the domain walls are behaved like invisible matter. When the vacuum energy density \(B_c\) is absorbed into the tension of the walls, we get \(P_q = \frac{1}{3} \rho_q\).
Using equations (4) and (15), we get the following expressions of quark energy density \( \rho_q \), quark pressure \( P_q \) and tension of the domain walls \( \sigma_d \)

\[
\rho_q = \frac{\left[8\pi + \mu\right](s-1)p' - 4\mu\theta + 8\pi s + 4\mu s - 8\pi\theta}{\gamma(64\pi^2 + 48\pi\mu + 8\mu^2)} t^{-2} - B_c,
\]

\[
P_q = \frac{\left[8\pi + \mu\right](s-1)p' - 4\mu\theta + 8\pi s + 4\mu s - 8\pi\theta}{\gamma(64\pi^2 + 48\pi\mu + 8\mu^2)} t^{-2} + B_c,
\]

\[
\sigma_d = \frac{\left[8\pi\gamma + \mu\gamma - 8\pi - 4\mu\right](s-1)p' - 4\pi s^2 - \mu\gamma s^2 + 4\mu\theta + }{8\pi(\gamma - 1)s + (\gamma - 4)\mu s - 4\pi(\gamma - 2)\theta}{\gamma(64\pi^2 + 48\pi\mu + 8\mu^2)} t^{-2}.
\]

From figure (2), it is observed that the quark energy density is decreasing function of time. It was large near the Big Bang and tends to zero at large time. The tension of the domain walls is negative which shows that the domain walls behave like invisible matter. Here, the graph is plotted for \( \gamma = 2 \) i.e. for stiff domain walls. From figure (1) and (2), it is found that the behavior of the quark energy density and tension of the domain walls is similar. So, in the case of strange quark matter, the walls interact with the primordial plasma and we also have stiff domain walls.
We discuss the stability of the model by using the function $c_s^2 = \frac{dP}{d\rho}$. The stability occurs when the function $c_s^2$ becomes positive [15]. One should note that the universe remain isotropic at large times only if the universe were stable against small anisotropic perturbations.

$$
\frac{dP}{d\rho} = \left[ \frac{3\mu(s-1)p' + 3\mu s + \mu s^2 + 4\pi s^2 - 4\mu \theta - 4\pi \theta}{8\pi + \mu(s-1)p' - 4\pi s^2 - \mu s^2 + 8\pi s + \mu s - 4\pi \theta} \right].
$$

(23)

From expression (23), it is clear that the function $c_s^2$ is constant and for particular choice of constant and other physical parameters, it becomes greater than zero. Hence, the model is stable.

The stability of the model is not time dependant in this model.

4. Model II

Here, we intend to consider the following functional form of $f(R,T)$.

$$
f(R,T) = f_1(R) + f_2(T) = R + bR^m + \mu T, \; b > 0, m > 0.
$$

(24)

In this functional form, we have considered $f_1(R) = R + bR^m$, $b > 0, m > 0$. The introduction of $R^m$ to the curvature term $R$ leads to two models: when $m > 1$, curvature becomes high whereas $m < 1$, we have low curvature [25]. Olmo [26] has studied model $f(R) = R \pm \lambda R^2$, where $\lambda$ is of the plank scale, in $f(R)$ theory of gravitation. The functional $f(R) = R + bR^m$ is widely studied in
\(f(R)\) theory of gravitation. So, an humble attempt has been made to introduce this new functional form in this theory.

The field equations (6) using equations (5), (8) and (24) explicitly written as

\[
\begin{align*}
\left[ p_1(s-1) - (s^2 - 2s + \theta) \right] t^{-2} - b(s^2 - 2s + \theta) t^{-2m} + 2bm(s^2 - 2s + \theta) t^{-2m-4} & = (16\pi + 5\mu)\rho + \mu\rho' \\
\left[ p_2(s-1) + (p_1 - 1)(2m + 2) + (2m + 2)2m(2m + 3) \right] t^{-2m-4} & = (16\pi + 5\mu)\rho + \mu\rho'
\end{align*}
\]

(25)

\[
\begin{align*}
\left[ p_2(s-1) - (s^2 - 2s + \theta) \right] t^{-2} - b(s^2 - 2s + \theta) t^{-2m} + 2bm(s^2 - 2s + \theta) t^{-2m-4} & = (16\pi + 5\mu)\rho + \mu\rho' \\
\left[ p_3(s-1) + (p_2 - 1)(2m + 2) + (2m + 2)2m(2m + 3) \right] t^{-2m-4} & = (16\pi + 5\mu)\rho + \mu\rho'
\end{align*}
\]

(26)

\[
\begin{align*}
\left[ p_3(s-1) - (s^2 - 2s + \theta) \right] t^{-2} - b(s^2 - 2s + \theta) t^{-2m} + 2bm(s^2 - 2s + \theta) t^{-2m-4} & = (16\pi + 5\mu)\rho + \mu\rho' \\
\left[ (s^2 - \theta) \right] t^{-2} - 2bm(s^2 - 2s + \theta) t^{-2m-4} & = (2s - 2\theta + 2m + 2) t^{-2m-4} + b(s^2 - 2s + \theta) t^{-2m} = (16\pi + \mu)\rho - 3\mu\rho
\end{align*}
\]

(28)

From equations (25)-(28), we yield

\[ p_1 = p_2 = p_3 = p' \]

(29)

Using equations (29) and equations (25)-(28), we obtain the energy density \(\rho\) and pressure \(P\) as follows

\[
\begin{align*}
\rho & = m_2 t^{-2} - m_4 t^{-2m} + m_4 t^{-2m-4}, \\
P & = m_3 t^{-2} + m_4 t^{-2m} + m_6 t^{-2m-4},
\end{align*}
\]

(30)

(31)

where

\[
\begin{align*}
m_1 & = 256\pi^2 + 96\pi\mu + 8\mu^2, m_3 = \frac{b(16\pi + 2\mu)(s^2 - 2s + \theta)}{m_1}, \\
m_2 & = \frac{1}{m_1} \left[ (32\pi + 2\mu)p'(s-1) - (16\pi + 2\mu)s^2 + (32\pi + 2\mu)s - 16\pi\theta \right], \\
m_4 & = \frac{2bm(s^2 - 2s + \theta)^{-1}}{m_1} \left[ (16\pi + \mu)p'(s-1) + (16\pi + \mu)(p'-1)(2m + 2) + (16\pi + \mu) \times \right] \\
& \left[ (2m + 2)^2 + 2m + 2\mu - 2m\mu - 2\mu \right], \\
m_6 & = \frac{1}{m_1} \left[ 6\mu\rho'(s-1) + (16\pi + 2\mu)s^2 + 6\mu s - 16\pi - 8\mu \right], m_6 = \frac{1}{m_1} \left[ (s^2 - 2s + \theta)^{-1} (16\pi + 2\mu) \right] \\
m_7 & = \frac{2bm(s^2 - 2s + \theta)^{-1}}{m_1} \left[ 3\mu\rho'(s-1) + 6\mu(m+1)(p'-1) + 6\mu(m+1)(2m+3) + \\
& (32\pi + 10\mu)s - (32\pi + 10\mu)\theta + 32m\pi + 10\mu n + 32\pi + 10\mu \right]
\end{align*}
\]
We observe that the energy density is decreasing function of time. From expression (30) it reveals that as \( t \to 0, \rho \to 0 \) and as \( t \to \infty, \rho \to 0 \). Comparison of the expressions (15) and (30), shows that there are negatively adding terms in this model. It might be effect of additional curvature term. In this section, the function \( c_i^2 = \frac{dP}{d\rho} \) is found to be

\[
\frac{dP}{d\rho} = \frac{2m_4 t^{-2} - 2mm_3 t^{-2m} + (2m + 4) m_4 t^{-2m - 4}}{2m_4 t^{-2} + 2mm_3 t^{-2m} + (2m + 4) m_4 t^{-2m - 4}}.
\] (32)

From figure (3), it is observed that the function \( c_i^2 \) is positive at the early stages of the evolution of the universe and tends to be negative at late time. Thus, the model is stable at the early phases of the universe.

Using equation (1) and (30), we obtain the following expressions of the quark energy density \( (\rho_q) \), quark pressure \( (P_q) \) and tension of the domain walls \( (\sigma_d) \)

\[
\rho_q = \frac{3(m_2 + m_5)}{4} t^{-2} + \frac{3(m_b - m_3)}{4} t^{-2m} + \frac{3(m_4 + m_2)}{4} t^{-2m - 4} - 2B_c,
\] (33)

\[
P_q = \frac{(m_2 + m_5)}{4} t^{-2} + \frac{(m_b - m_3)}{4} t^{-2m} + \frac{(m_4 + m_2)}{4} t^{-2m - 4} - \frac{2}{3} B_c,
\] (34)

\[
\sigma_d = \frac{(m_2 - 3m_5)}{4} t^{-2} - \frac{(3m_b + m_3)}{4} t^{-2m} + \frac{(m_4 - 3m_5)}{4} t^{-2m - 4} + B_c.
\] (35)

**Fig.3** Plot of the function \( c_i^2 = \frac{dP}{d\rho} \) with cosmic time.
It is clear from figure (4) that the quark energy density and the tension of the domain walls are decreasing functions of cosmic time. They are positive and large near the Big Bang singularity. It tends to be zero at large time. Thus, the domain walls are present in the early stages of the evolution of the universe and disappear at large time which is in accordance with Zel’dovich et al. [18]. When the vacuum energy density $B_c$ is absorbed into the tension of the walls, we get

$$P_q = \frac{1}{3} \rho_q.$$ 

![Fig. 4. Plot of quark energy density and tension of the domain walls with cosmic time.](image)

Using equations (4) and (30), we get the following expressions of the quark energy density ($\rho_q$), quark pressure ($P_q$) and tension of the domain walls ($\sigma_d$)

$$\rho_q = \frac{(m_2 + m_s)}{\gamma} t^{-2} + \frac{(m_6 - m_s)}{\gamma} t^{-2m} + \frac{(m_4 + m_7)}{\gamma} t^{-2m-4} - B_c,$$

$$P_q = \frac{(\gamma - 1)(m_2 + m_s)}{\gamma} t^{-2} + \frac{(\gamma - 1)(m_6 - m_s)}{\gamma} t^{-2m} + \frac{(\gamma - 1)(m_4 + m_7)}{\gamma} t^{-2m-4} + B_c,$$

$$\sigma_d = \frac{(m_2(\gamma - 1) - m_s)}{\gamma} t^{-2} + \frac{(-m_6 + (1-\gamma)m_7)}{\gamma} t^{-2m} + \frac{(m_4(\gamma - 1) - m_7)}{\gamma} t^{-2m-4}.$$ 

From figure (5), it reveals that the quark energy density is decreasing function of cosmic time. It is large near the Big Bang singularity. It tends to be zero at large time. Also, the tension of the domain walls is decreasing function of cosmic time. It is positive and large near the Big Bang. It tends to be zero at large time. Thus, the domain walls are present in the early stages of the evolution of the universe and disappear at large time which is in accordance with Zeldovich et al. [18]. In this model, the graph is plotted for $\gamma = 2$ i.e. for stiff domain walls. It is noted from
figure (4) and (5) that the behavior of the quark energy density and tension of the domain walls similar. So, in the case of strange quark matter of the model also the walls witness interaction with the primordial plasma and we have stiff domain walls.

![Quark energy density and tension of the domain walls with cosmic time](image)

**Fig.5.** Plot of the quark energy density and tension of the domain walls with cosmic time.

In section 3, we find that domain wall behave like invisible matter where as in this section they are presents in the early universe. This shows the effect of curvature. Also, one can note that the stability of the model affected due to curvature.

**5. Model III**

In this section, we are presenting model $\Lambda(T)$ which is now widely studied in the literature of $f(R,T)$ theory of gravitation. It has the following form

$$f(R,T) = \lambda R + \lambda T,$$ (39)

The field equations (6) using equation (39) reduces to the following form

$$R_{ij} - \frac{1}{2} \, R g_{ij} = \left( \frac{8\pi + \lambda}{\lambda} \right) T_{ij} + \left( p + \frac{1}{2} T \right) g_{ij}.$$ (40)

The Einstein field equations with cosmological constant term have the following form

$$R_{ij} - \frac{1}{2} \, R g_{ij} = -8\pi T_{ij} + \Lambda g_{ij}.$$ (41)

Equation (40) and (41) gives us
\[ \Lambda = \Lambda(T) = P + \frac{1}{2} T \quad (42) \]

and \(-8\pi = \left( \frac{8\pi + \lambda}{\lambda} \right)\). From expression (42), it is observed that the cosmological constant term is depends on the trace of the energy momentum tensor \(T\). This type of the model is called as \(\Lambda(T)\) gravity model. Poplawski [16] has proposed this model where the cosmological constant in the gravitational Lagrangian is considered as a function of the trace energy momentum tensor. Sahoo et al. [17] have considered the Kaluza-Klein cosmological model in the modified gravity with \(\Lambda(T)\).

The field equations (40) for the metric (5) and using equation (6),(39) reduces to

\[
\left[ p_1(s-1) - \frac{1}{2} \left( s^2 - 2s + \theta \right) \right] t^{-2} = \left( \frac{8\pi + \lambda}{\lambda} \right) \rho + \Lambda, \quad (43)
\]

\[
\left[ p_2(s-1) - \frac{1}{2} \left( s^2 - 2s + \theta \right) \right] t^{-2} = \left( \frac{8\pi + \lambda}{\lambda} \right) \rho + \Lambda, \quad (44)
\]

\[
\left[ p_3(s-1) - \frac{1}{2} \left( s^2 - 2s + \theta \right) \right] t^{-2} = \left( \frac{8\pi + \lambda}{\lambda} \right) \rho + \Lambda, \quad (45)
\]

\[
\left[ \frac{1}{2} (\theta - s^2) \right] t^{-2} = \left( \frac{8\pi + \lambda}{\lambda} \right) P + \Lambda. \quad (46)
\]

Equations (10)-(11) yields

\[ p_1 = p_2 = p_3 = p'. \quad (47) \]

Using equations (47) and equations (43)-(46), we obtain the energy density (\(\rho\)), pressure (\(P\)) and (\(\Lambda\)) as follows

\[
\rho = \frac{\lambda \left[ 4(16\pi + \lambda)(s-1)p' - (32\pi + 3\lambda)s^2 + 4(16\pi + \lambda)s - (32\pi + \lambda)\theta \right]}{512\pi^2 + 192\pi \mu + 13\lambda^2} t^{-2}, \quad (48)
\]

\[
P = \frac{\lambda \left[ 6\lambda(s-1)p' + (16\pi + 2\lambda)s^2 + 6\lambda s - (16\pi + 8\lambda)\theta \right]}{512\pi^2 + 192\pi \mu + 13\lambda^2} t^{-2}, \quad (49)
\]

\[
\Lambda = \frac{\lambda \left[ 48\pi + 2\lambda(s-1)p' - (80\pi + 7\lambda)s^2 + (192\pi + 18\lambda)s - (112\pi + 11\lambda)\theta \right]}{1024\pi^2 + 384\pi \mu + 26\lambda^2} t^{-2}. \quad (50)
\]

Here, energy density is decreasing function of cosmic time. The nature of the energy density is same as in the section 3.
The function \( c_s^2 = \frac{dP}{d\rho} \) in this section is obtained as

\[
\frac{dP}{d\rho} = \left[ \frac{6\lambda(s-1)p' + (16\pi + 2\lambda)s^2 + 6\lambda s - (16\pi + 8\lambda)\theta}{4(16\pi + \lambda)(s-1)p' - (32\pi + 3\lambda)s^2 + 4(16\pi + \lambda)s - (32\pi + \lambda)\theta} \right].
\] (51)

From expression (51), it is clear that the function \( c_s^2 \) is constant and for particular choice of constant and other physical parameters, it becomes greater than zero. Hence, the model is stable.

Using equation (1) and (48), we found the following expressions of the quark energy density \( (\rho_q) \), quark pressure \( (P_q) \) and tension of the domain walls \( (\sigma_d) \)

\[
\rho_q = \frac{3\lambda\left[ (64\pi + 10\lambda)(s-1)p' - (16\pi + \lambda)s^2 + (64\pi + 10\lambda)s - (48\pi + 9\lambda)\theta \right]}{4(512\pi^2 + 192\pi\mu + 13\lambda^2)} t^{-2} - B_c,
\] (52)

\[
P_q = \frac{\lambda\left[ (64\pi + 10\lambda)(s-1)p' - (16\pi + \lambda)s^2 + (64\pi + 10\lambda)s - (48\pi + 9\lambda)\theta \right]}{4(512\pi^2 + 192\pi\mu + 13\lambda^2)} t^{-2} - \frac{2}{3} B_c,
\] (53)

\[
\sigma_d = \frac{\lambda\left[ (64\pi - 14\lambda)(s-1)p' - (80\pi + 9\lambda)s^2 + (64\pi - 14\lambda)s + (16\pi + 23\lambda)\theta \right]}{4(512\pi^2 + 192\pi\mu + 13\lambda^2)} t^{-2} + B_c.
\] (54)

From figure (6), it is found that the tension of the domain walls is negative. Thus, the domain walls behave like invisible matter. When the vacuum energy density \( B_c \) is absorbed into the tension of the walls, we get \( P_q = \frac{1}{3} \rho_q \). The \( \Lambda \) is positive and decreasing function of the cosmic time which is consistent with the lambda cold dark matter. The result is similar to the result obtained by Sahoo et al. [17]. Also, the quark energy density is decreasing function of the cosmic time and tends to be zero at large time.
Fig.6. Plot of the quark energy density, tension of the domain walls, Lambda with cosmic time.

Using equation (4) and equation (48) we get the following expressions of the quark energy density ($\rho_q$), quark pressure ($P_q$) and tension of the domain walls ($\sigma_d$)

$$\rho_q = \frac{\lambda [(64 \pi + 10 \lambda) (s - 1) p' - (16 \pi + \lambda) s^2 + (64 \pi + 10 \lambda) s - (48 \pi + 9 \lambda) \theta]}{\gamma (512 \pi^2 + 192 \pi \mu + 13 \lambda^2)} t^{-2} - B_c,$$

$$P_q = \frac{(\gamma - 1) \lambda [(64 \pi + 10 \lambda) (s - 1) p' - (16 \pi + \lambda) s^2 + (64 \pi + 10 \lambda) s - (48 \pi + 9 \lambda) \theta]}{\gamma (512 \pi^2 + 192 \pi \mu + 13 \lambda^2)} t^{-2} + B_c,$$

$$\sigma_d = \frac{\lambda [(64 \pi \gamma + 4 \lambda \gamma - 64 \pi - 10 \lambda) (s - 1) p' + (16 \pi + \lambda - 32 \pi \gamma - 3 \lambda \gamma) s^2 + \lambda [(64 \pi \gamma + 4 \lambda \gamma - 64 \pi - 10 \lambda) s + (48 \pi + 9 \lambda - 32 \pi \gamma - \lambda \gamma) \theta]}{\gamma (512 \pi^2 + 192 \pi \mu + 13 \lambda^2)} t^{-2}.$$

From figure (7), it is observed that the quark energy density is decreasing function of time. It was large near the Big Bang and tends to zero at large time. The tension of the domain walls is negative which shows that the domain walls behave like invisible matter. In this model, the graph is plotted for $\gamma = 2$ i.e. for stiff domain walls. We can see that the behavior of the quark energy density and the tension of the domain walls in figure (6) and (7) are alike. So, in this case also the walls are found to be interacting with the primordial plasma and we have stiff domain walls in case of strange quark matter of the model.
The volume of the models is defined as

\[ V = t^4. \]  

(58)

The expansion scalar is obtained as

\[ \theta = \frac{V_t}{V} = \frac{3p'}{t}. \]  

(59)

The deceleration parameter is found to be

\[ q = \frac{d}{dt} \left( \frac{1}{H} \right) - 1 = \frac{1 - p'}{p'}. \]  

(60)

The volume is increasing function of the cosmic time. The expansion scalar is decreasing function of the cosmic time. It is large near the Big Bang. Thus, the universe is expanding and the rate of expansion of the universe is decreasing with increasing time. The sign of the deceleration parameter indicates whether the model is accelerating or decelerating. The positive sign corresponds to the acceleration of the universe whereas the negative sign of the deceleration parameter corresponds to the deceleration of the universe. Here, the deceleration parameter is negative for \( p' > 1 \) i.e. the universe is accelerating for \( p' > 1 \).

6. Conclusion

In this paper, we have explored the strange quark matter coupled to the domain walls in the Bianchi type-I Kasner universe in the context of the \( f(R,T) \) modified theory of gravitation. We have discussed three different functional forms of the function \( f(R,T) \). We found that the
volume of the universe is increasing. The universe is expanding and the expansion rate is decreasing. The universe is accelerating for \( p' > 1 \). The quark energy density is decreasing function of the cosmic time. We have discussed three functional form of \( f(R, T) \). For the functional forms \( R + 2\mu T \) and \( \lambda R + \lambda T \), we get negative tension of the domain walls. For the functional form \( R + bR^m + \mu T \), we observed that the domain walls present at the early stages and vanish at the present which is in accordance with Zeldovich et al. [18]. It is remarkable that the curvature affect on the behavior of the matter source as well as stability of the model. We also noted that in the all models we do not have gravitational collapse which we have observed in our earlier work on domain walls [27, 28]

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